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A method for characterisation of the static elastic properties of the porous frame of orthotropic open-cell foams

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ABSTRACT

This paper proposes a method to identify the static, fully relaxed elastic Hooke's matrix of a porous open-cell material. The moduli are estimated through an inverse estimation method, by performing a fit of a numerical model on the measured displacements on the faces of the porous material. These displacements are obtained from a static compression along each of the three coordinate axes. The material is modelled as an orthotropic equivalent solid, of which the principal directions are not necessarily aligned with the orthonormal coordinate system in which the experiments are conducted. The angles of relative orientation accounting for the misalignment are among the properties to be estimated. The focus in this paper is on the methodology itself, and its validity is verified by applying the method to four artificial materials with different levels of anisotropy. In addition, the robustness of the method to perturbations on the input data is investigated.

1. Introduction

The properties of lightweight open-cell porous materials for acoustic applications are highly dependent on the production process. The production process itself influences the micro-structure and geometry of the material, possibly inducing anisotropy in the macroscopic properties of the material. At present, the degree of anisotropy in the porous material due to the production process is unclear, and the implications of the anisotropy are not yet fully understood.

The influence of anisotropy in porous materials on the predicted acoustic performance of multi-layered arrangements has been studied and reported as significant by Göransson and Hörlin (2010), Hörlin and Göransson (2010) and Lind Nordgren, Göransson, Deü, and Dazel (2013). In order to confirm the results and to assess these effects in a real foam, the degree of anisotropy, together with the corresponding anisotropic properties of a material, needs to be determined.

Several authors have attempted the characterisation of the anisotropic properties of porous materials, often assuming that the material exhibits a certain symmetry, and that the principal material directions are aligned with the imposed coordinate axes. Jaouen, Renault, and Deverge (2008) give an overview of the available experimental methods for elastic and damping characterisation of acoustical porous materials. The studied methods are all under quasi-static or dynamic excitation of the material, and only the method described by Melon, Mariez, Ayrault, and Sahraoui (1998) allows for characterisation of transversely isotropic materials, assuming that the principal material directions are known.

In addition, most real porous materials are exhibiting an anelastic behaviour, since the predominant component of porous materials is usually an elastomer for which the rubbery regime often spans room temperature (Gibson & Ashby, 1997). A

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method for characterisation of the elastic and anelastic properties of open-cell foams was recently proposed by Cuenca and Göransson (2012) and applied to melamine foam by Cuenca, Van der Kelen, and Göransson (2014). The reported method uses the augmented Hooke's law (Dovstam, 2000), separating the stiffness matrix into a static, elastic matrix and a frequency-dependent, anelastic matrix. This separation was motivated by Biot (1954) who introduced the concept of hidden thermodynamic variables, and described the dynamic elastic moduli of the solid as a superposition of elastic and anelastic contributions. The focus of the current paper is on the full characterisation of the anisotropic static elastic properties of a porous material, which has not been attempted yet to the knowledge of the authors.

The present work is an important step towards the generalisation of the work of Cuenca and Göransson (2012) and Cuenca et al. (2014), where one of the assumptions made was that the elastic and anelastic moduli are collinear. The method proposed in this paper allows to remove this restriction, as it provides the fully relaxed, elastic properties independently, and the same material can be characterised using both methods, with the resulting static elastic matrix and the frequency-dependent anelastic matrices possibly having different degrees of anisotropy. The possibility of these matrices possessing different symmetries was already reported by Dovstam (2000) and Biot (1954). Whether or not this is the case for the elastomers that constitute the solid frame of porous foams, is an open question.

The main contribution of this paper is to propose a method for the characterisation of the anisotropic static elastic properties of a porous material. As a proof of concept concerning the feasibility of the inverse estimation approach proposed, numerically simulated measurements are used as targets in the inverse estimation. The current paper starts with the material model and the characterisation method, which is then applied to four fictitious materials, one isotropic, one transversely isotropic, one orthotropic and one anisotropic material sample, to verify the method and its applicability for the present purpose. A study of the numerical robustness of the method is included as well.

2. Material model

Biot (1956) described the constitutive laws for porous materials, which consist of a solid phase and a fluid phase, most frequently air in acoustic applications. In the case of open-cell porous materials, the fluid phase only takes part in the dynamic deformation (Melon et al., 1998). Therefore, the static structural properties of the material, such as elasticity, can conveniently be described in the absence of the fluid phase. Then, only the solid frame of the material is here considered and the Hooke's law for the material may be written as

$$\underline{\boldsymbol{\sigma}}(\omega) = \mathbf{H}(\omega)\underline{\boldsymbol{\varepsilon}}(\omega), \tag{1}$$

giving the relation between the frequency-dependent stresses and strains in the material in vacuum, where $\mathbf{H}(\omega)$ is the stiffness or Hooke's matrix. For the notation of the stresses and strains, the convention used is

$$\begin{aligned} \underline{\boldsymbol{\sigma}} &= [\,\sigma_{11} \quad \sigma_{22} \quad \sigma_{33} \quad \sigma_{23} \quad \sigma_{31} \quad \sigma_{12}\,]^{\mathrm{T}}, \\ \underline{\boldsymbol{\varepsilon}} &= [\,\varepsilon_{11} \quad \varepsilon_{22} \quad \varepsilon_{33} \quad 2\varepsilon_{23} \quad 2\varepsilon_{31} \quad 2\varepsilon_{12}\,]^{\mathrm{T}}. \end{aligned} \tag{2}$$

The stiffness matrix $\mathbf{H}(\omega)$ may be rewritten according to the augmented Hooke's law (Dovstam, 2000) as a superposition of an elastic, frequency-independent part describing the fully relaxed material deformation, and an anelastic, frequency-dependent part for the reversible visco-elastic deformation,

$$\mathbf{H}(\omega) = \mathbf{H}^{(0)} + \tilde{\mathbf{H}}(\omega). \tag{3}$$

This paper focuses on the characterisation of the static, frequency-independent part $\mathbf{H}^{(0)}$ of the Hooke's matrix. A proof of concept for estimation of the frequency-dependent part $\tilde{\mathbf{H}}(\omega)$ is given by Cuenca and Göransson (2012), and an application to a melamine foam in Cuenca et al. (2014), in which the material should be placed in vacuum conditions during the experiment to extract the influence of air. For the static part of the Hooke's matrix discussed here, the material needs not be in vacuum (Hörlin & Göransson, 2010), given that the deformation is slow enough. As no pressure variation is present in the material, it may be modelled as an equivalent solid material.

The anisotropic Hooke's matrix $\mathbf{H}_{\mathrm{a}}^{(0)}$ for an orthotropic material, presented in a coordinate system that is not necessarily aligned with the natural coordinate system is given as

$$\mathbf{H}_{\mathrm{a}}^{(0)} = \begin{bmatrix} H_{\mathrm{a},11}^{(0)} & H_{\mathrm{a},12}^{(0)} & H_{\mathrm{a},13}^{(0)} & H_{\mathrm{a},14}^{(0)} & H_{\mathrm{a},15}^{(0)} & H_{\mathrm{a},16}^{(0)} \\ & H_{\mathrm{a},22}^{(0)} & H_{\mathrm{a},23}^{(0)} & H_{\mathrm{a},24}^{(0)} & H_{\mathrm{a},25}^{(0)} & H_{\mathrm{a},26}^{(0)} \\ & & H_{\mathrm{a},33}^{(0)} & H_{\mathrm{a},34}^{(0)} & H_{\mathrm{a},35}^{(0)} & H_{\mathrm{a},36}^{(0)} \\ & & & H_{\mathrm{a},44}^{(0)} & H_{\mathrm{a},45}^{(0)} & H_{\mathrm{a},46}^{(0)} \\ & \text{(sym)} & & & H_{\mathrm{a},55}^{(0)} & H_{\mathrm{a},56}^{(0)} \\ & & & & & H_{\mathrm{a},66}^{(0)} \end{bmatrix}. \tag{4}$$