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Nonlinear dynamics of microplates

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ABSTRACT

In this paper, the nonlinear dynamics of a microplate is investigated based on the modified couple stress theory. The von Kármán plate theory is employed to model the system by retaining in-plane displacements and inertia. The equations of motion are derived via an energy method based on the Lagrange equations, yielding a set of second-order nonlinear ordinary differential equations with coupled terms. These equations are recast into a set of first-order nonlinear ordinary differential equations and the resulting equations are solved by means of the pseudo-arclength continuation technique. The nonlinear dynamics is examined through plotting the frequency-response and force-response curves of the system. The influence of system parameters on the resonant responses is highlighted. The differences in the response amplitude of the system modelled based on the modified couple stress theory and the classical one are discussed.

1. Introduction

Continuous elements of microscale dimensions (Farokhi, Ghayesh, & Amabili, 2013a; Ghayesh, Amabili, & Farokhi, 2013a; Ghayesh, Farokhi, & Amabili, 2013b; Ghayesh, Farokhi, & Amabili, 2014) are widely found in mechanical and bio engineering. Microplates, among them, are used in many applications such as in biosensors, biomechanical organs, microswitches, microactuators, and vibration and shock sensors. Experimental investigations (Fleck, Muller, Ashby, & Hutchinson, 1994; McFarland & Colton, 2005) revealed that microscale structures display strange size-dependent deformation behaviour which cannot be predicted by classical continuum mechanics theories. With the advent of new continuum mechanics theories such as the strain gradient (Ghayesh, Amabili, & Farokhi, 2013) and modified couple stress theories, a great deal of research has been carried out to predict the size-dependent behaviour theoretically (Farokhi, Ghayesh, & Amabili, 2013b; Ghayesh, Farokhi, & Amabili, 2013). The literature regarding the dynamics and statics of microplates can be grouped into two general classes, i.e., *linear* and *nonlinear*.

The literature regarding the *linear* size-dependent behaviour of microplates is quite large. Reviewing some recent papers, for example, Lazopoulos (Lazopoulos, 2009) examined the linear bending of thin microplates based on the strain gradient elasticity; he took into account the surface energy and employed a variational method to derive the equations of motion. Wang, Zhou, Zhao, and Chen (2011) employed the strain gradient elasticity theory to develop a size-dependent microplate model by means of the Kirchhoff theory; the model developed in this paper is linear with ignoring in-plane displacements. Jomehzadeh, Noori, and Saidi (2011) employed the modified couple stress theory to derive the linear out-of-plane equation of motion of a microplate and conduct a vibration analysis. Hashemi and Samaei (2011) investigated the linear buckling

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behaviour of micro/nonoplates subjected to in-plane excitation loads; the analysis is based on the nonlocal Mindlin plate theory. Farajpour, Shahidi, Mohammadi, and Mahzoon (2012) analyzed the linear buckling response of an orthotropic single-layered graphene sheet by means of a nonlocal elasticity; they employed the nonlocal theory of Eringen to derive the linear equations of motion. Nabian, Rezazadeh, Almassi, and Borgheei (2013) investigated the stability characteristics of a functionally graded microplate subjected to hydrostatic and electrostatic pressure. Roque, Ferreira, and Reddy (2013) obtained the bending response of first-order shear deformable microplates based on the modified couple stress theory and a meshless method. Ramezani (2012) contributed to the field by developing a first-order shear deformation microplate model on the basis of the strain gradient elasticity theory. Ashoori Movassagh and Mahmoodi (2013) obtained the linear equation governing the out-of-plane dynamical behaviour of a microplate based on the modified strain-gradient elasticity theory; they employed the extended Kantorovich method (EKM) to obtain the transverse vibrations of the microplate. Li, Zhou, Zhou, and Wang (2014) developed a size-dependent linear model for bi-layered Kirchhoff microplate motions based on the modified couple stress theory; they obtained the bending behaviour of a simply supported bi-layered square microplate subjected to a constantly distributed load.

The literature regarding the *nonlinear* dynamics of microplates, categorized in the second group, is not large. For example, Asghari (2012) developed the size-dependent equations of motion of microplates based on the modified couple stress theory. Thai and Choi (2013) developed a size-dependent model of functionally graded Kirchhoff and Mindlin plates on the basis of the modified couple stress theory. In both of the aforementioned valuable studies, only the equations of motion were derived and no solutions for the *nonlinear dynamics* were provided.

In this paper, for the first time, the *nonlinear forced dynamics* of a microplate subjected to a distributed harmonic excitation load is examined by constructing frequency-response and force-response curves. The microplate is modelled based on the modified couple stress theory through use of the von Kármán plate theory taking into account Kirchhoff's hypotheses, retaining all *in-plane* and *out-of-plane* displacements and inertia; it is the first time that all the in-plane and out-of-plane displacements and inertia are retained in the nonlinear analysis of microplates – this operation is numerically expensive and the computer codes should be well-optimized for accuracy and run-time. The size-dependent potential energy and the kinetic energy of the system are constructed as functions of the displacement field and inserted in the Lagrange equations yielding a set of second-order nonlinear ordinary differential equations with coupled terms – it is also the first time that the equations of motion are developed via the Lagrange equations. These equations are transformed into a set of first-order nonlinear ordinary differential equations via a change of variables and then solved by means of the pseudo-arc-length continuation technique; a discretized system with a large number of degrees of freedom is employed to capture almost all modal interactions and to obtain accurate results. Frequency-response and force-response curves of the system are plotted for different system parameters with emphasis on stable and unstable solution branches and bifurcation points. The importance of taking into account the length-scale parameter is also highlighted.

It is known that in case of large-amplitude deformations, the linear theory fails to predict the response of the system accurately, since the contributions of the nonlinear terms in the equations of motion increase significantly. On the other hand, in many applications, such as resonators (Ghayesh, Farokhi, & Amabili, 2013), the microplate undergoes large-amplitude deformations (of the order of thickness) and hence its dynamical behaviour, especially in the resonant regime, is largely affected by the presence of both cubic and quadratic nonlinear terms in the equations of motion, caused by assuming nonlinear strain–displacement relations. Hence, in order to examine the dynamic response of a microplate undergoing large deformations, it is necessary to employ a nonlinear model to be able to obtain reliable results.

2. Model development and solution method

Shown in Fig. 1 is a rectangular microplate with in-plane dimensions a and b in the x and y directions, respectively, and thickness h. The displacement field of the microplate is defined in a Cartesian coordinate system (O; x, y, z) with the origin O

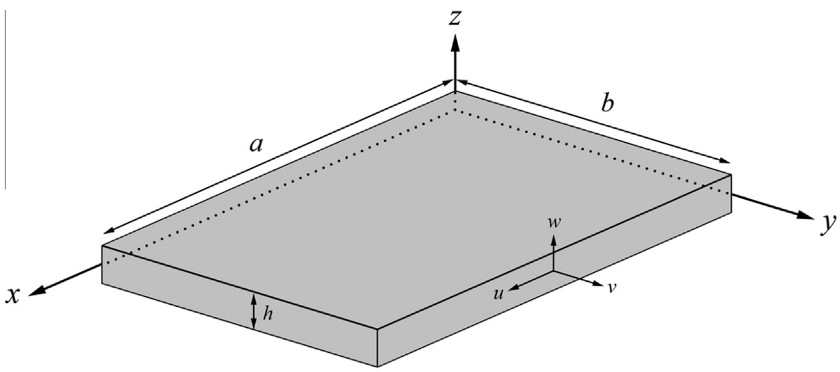

Fig. 1. Schematic representation of a rectangular microplate.