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The strong and weak forms of a general imperfect interface model for linear coupled multifield phenomena

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ABSTRACT

Imperfect interfaces are present in a variety of practical situations in mechanics and physics. In particular, they play an essential role in determination of mechanical and physical properties of composite materials. In a recent paper by Gu and He (2011), a general imperfect interface model for linear coupled multifield phenomena is rigorously derived which includes as special ones the widely used linear imperfect interface models proposed on the basis of some phenomenological considerations. The present paper aims at the establishment of the weak form for the boundary value problem of composites with general imperfect interfaces, which is the first key step towards numerically implementing the imperfect interface effects within the framework of finite element. To achieve this objective, firstly, the general imperfect interface model is reformulated in such a way that a particularly simple and compact strong form is obtained for it. Then, its weak form is derived in the general context of coupled multifield phenomena and expressed in a unified way for various linear mechanical and physical phenomena. Finally, the general results are particularized to transport phenomena, elasticity and piezoelectricity, together with detailed discussions on extreme situations in each case.

1. Introduction

In general, the prediction of the effective properties of a composite based on the hypotheses of the perfect interfaces ignores the interface effects between constituents in composite. However, in practice, interfaces in composite exhibit properties quite different from those associated with the surrounding constituents and significantly affect the macroscopic properties of composite. Recently, studies on interface phenomena within heterogeneous materials has been stimulated and accelerated considerably by the rapid development of nanomaterials. This is attributed to the large interface-to-volume ratio for nanomaterials, which makes the interface effects especially significant and leads to size-dependent overall properties (see, e.g., Duan, Wang, Huang, & Karihaloo, 2005; Duan, Wang, Huang, & Luo, 2005; Duan, Wang, & Karihaloo, 2009).

Various interface models have been proposed by different approaches for different mechanical and physical problems. Referring to the single phenomena such as linear elasticity, the most widely used ones are the spring-layer interface model and the coherent interface model. In the spring-layer interface model which is the most spread one in engineering (see, e.g.,

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Achenbach & Zhu, 1989; Benveniste, 1985; Hashin, 1990, 1991, 1992, 2002; Qu, 1993), the traction vector is continuous across an interface and proportional to the jump of the displacement vector across the same interface. In the coherent interface model first proposed by Gurtin and Murdoch (1975) which is the one commonly adopted for modeling the surface/interface effects in nanomaterials and nano-sized structures (see, e.g., Chen & Dvorak, 2006; Chen, Dvorak, & Yu, 2007a, Chen, Dvorak, & Yu, 2007b; Dingreville, Qu, & Cherkaoui, 2005; Javili & Steinmann, 2009, 2010, 2011; Shenoy, 2002; Sharma, Ganti, & Bhate, 2003; Sharma & Ganti, 2004; Le-Quang & He, 2007, 2008, 2009; Mitrushchenkov, Chambaud, Yvonnet, & He, 2010; Mogilevskaya, Crouch, & Stolarski, 2008; Mogilevskaya, Crouch, Stolarski, & Benusiglio, 2010; Yvonnet, Mitrushchenkov, Chambaud, & He, 2011), the displacement vector is continuous across an interface while the traction field across the same interface is discontinuous and has to verify the Laplace–Young equation. In particular, it is shown that the aforementioned two types of imperfect interface models can be associated to the extreme cases of thin soft and stiff interphases of the general elastic interface model proposed by Benveniste (2006), which is characterized by the traction and displacement jump relations to be satisfied by an interface replacing an interphase of small uniform thickness between two adjacent phases. In the setting of linear thermal conduction, the counterparts of the spring-layer and membrane-type imperfect interface models are Kapitza's resistance model and the highly conducting interface model, which are used in a great number of works (see, e.g., Benveniste, 1986, 1987; Hashin, 2001; Lipton, 1986). In particular, it is shown that these two imperfect interface models can be recovered as special cases of the general one (Benveniste, 2006; Gu, 2008).

For the coupled phenomena such as linear piezoelectricity, Chen (2008) proposed the piezoelectric surface/interface model which can be considered as the generalization of the coherent interface model to the piezoelectricity. According to this model, the displacement and the electric field are continuous across the interface, while the traction vector and the normal electric displacement undergo jumps on the interface which are proportional to certain surface differential operators of the displacement and the electric field, respectively. This model has been widely applied in studying the size-dependent effective piezoelectric moduli of nanocomposites (see, e.g., Xiao, Xu, & Zhang, 2001). The piezoelectric spring-type interface states that the traction vector field and the normal electric displacement are discontinuous across the interface, while the displacement and the electric field undergo jumps on the interface which are proportional to the traction vector and the normal electric displacement field. The effect of piezoelectric spring-type interface in piezoelectric composites has been investigated (see, e.g., Fang, Liu, Jin, & Wen, 2009; Wang & Sudak, 2007; Wang & Pan, 2010). Recently, by combining the establishment of the coordinate-free interfacial operators and the idea of interphase replacement inspired from those proposed by Bövik (1994) and Benveniste (2006, 2009), Gu and He (2011) and Gu (2008) formulated the coordinate-free general interface model for all the linear phenomena in a unified manner. In additions, initiated from Benveniste (2006) to treat the single phenomena, (Gu, 2008) obtained four special piezoelectric interface models including the aforementioned two special piezoelectric surface/interface model by taking interphase be the corresponding extreme.

To numerically simulate surface and interface effects inside materials and structures, a large number of methods have been proposed (see, e.g., Fries & Belytschko, 2010; Fries, 2009; Hansbo & Hansbo, 2002, 2004; Javili, McBride, & Steinmann, 2012, 2013). In our opinion, among all these methods, the most promising one is that combing the level-set method and extended finite element method (XFEM) initially proposed by Belytschko and Black (1999) and Moës, Dolbow, and Belytschko (1999) to deal with cracks problem due to the minimal remeshing properties. From available literatures, it is seen that significant progress has been made in the treatment of the interface effects inside composite materials within the framework of extended finite element method (XFEM). For example, Yvonnet, He, and Toulemonde (2008); Yvonnet, He, Zhu, and Shao, 2010 applied the XFEM to study the effect of the highly and weakly conducting interfaces on the macroscopic properties of the composite, respectively; Yvonnet, Le-Quang, and He (2008) numerically simulated the effect of the coherent interface model in composite to compute the size-dependent effective properties of nanocomposites; Zhu, Gu, Yvonnet, Shao, and He (2011) and Zhu (2012) elaborated an efficient numerical approach for dealing with three-dimensional linear spring-layer curved interfaces. As previously discussed, those works only dealt with specific models merely associated to the extreme case of the general interface. The general thermal imperfect interface model is much more complicated than the special ones. Taking the general thermal imperfect interface as an example, it involves both a temperature jump and a normal heat flux jump across an interface. In the treatment of the general interface, Harari and Dolbow (2010), Farsad, Vernerey, and Park (2010) developed an numerical approach to investigate the interface effect separately within thermal and elastic fields. However, the jump conditions on interfaces treated in those works are lack of physical interpretation. This type of the general interface model can not reduce to the special cases by pushing the interphase parameters to the extremity, such as the spring-layer model and coherent interface model for elasticity, the Kapitza's interfacial thermal resistance model and the highly conducting interface model for thermal problem. In addition, to the best of the authors' knowledge, XFEM has not been used to implement the general interface model for the linear coupled phenomena. Thus, the investigation to numerical simulate the imperfect interface phenomena remains largely open.

According to both theoretical and numerical experiences, it appears that key steps towards simulating numerically the interface effects inside composite materials in the framework of XFEM are the establishment of the weak formulation of the relevant boundary value problem, the discretization of this problem by using suitable enrichment functions, and the detection of cut elements on basis of the level-set method (see, e.g., Yvonnet et al., 2008 Yvonnet & Le-Quang et al., 2008; Yvonnet et al., 2010). We take the elastic interface problem as an example for constructing the XFEM methodology (see, e.g., Zhu et al., 2011). In this case, to elaborate an XFEM for dealing with the interface problem, the level-set method is firstly applied to detect the cut elements. Then, according to the type of imperfect interface, an approximation of the displacement in the cut element is proposed where the suitable discontinuous enrichment function is defined for simulating