International Journal of Engineering Science 85 (2014) 66–73

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Short Communication

A discussion on different non-classical constitutive models of microbeam

Amir Mehdi Dehrouyeh-Semnani *

School of Mechanical Engineering, College of Engineering, University of Tehran, Iran

ARTICLE INFO

Article history:
Received 25 June 2014
Received in revised form 16 July 2014
Accepted 24 July 2014
Available online 14 September 2014

Keywords:
Microbeam
Euler–Bernoulli beam model
Constitutive beam model
Size effect
Modified couple stress theory
Modified strain gradient elasticity theory

ABSTRACT

In this study, static bending of thin plane-strain microbeam is investigated based on modified couple stress and modified strain gradient elasticity constitutive beam models. It is indicated that the results are obtained based on modified couple stress constitutive beam model are in excellent agreement with those observed experimentally, like modified strain gradient elasticity one. Comparison between the results of the constitutive beam models for static bending behavior with common boundary conditions (i.e. clamped-free, clamped–clamped, clamped–pined and pined–pined) reveals that these beam models are in very good agreement. In addition, the constitutive beam models are compared with modified couple stress and modified strain gradient elasticity Euler–Bernoulli beam models.

1. Introduction

Yang, Chong, Lam, and Tong (2002) modified couple stress theory by introducing an additional equilibrium relation to govern the behavior of couples. On the basis of this modification, a linear elastic model for isotropic materials was developed. By the same way, (Lam, Yang, Chong, Wang, & Tong, 2003) modified strain gradient elasticity theory. Constitutive relations of modified strain gradient elasticity were employed to develop governing equation and associated boundary conditions of microbeam. In addition, by comparison of the results of the new beam model with the experimental data, it was indicated that the new beam model validates the experimental data very well. Park and Gao (2006) proposed modified couple stress Euler–Bernoulli beam model and they indicated that the new beam results agree fairly well with the experimental data reported by Lam et al. (2003). Kong, Zhou, Nie, and Wang (2009) developed modified strain gradient elasticity Euler–Bernoulli beam model. It is notable that the governing equation of microbeam was modified by Akgöz and Civalek (2012b). Kong et al. (2009) indicated that the new beam model predicts lower values for static deflection than those obtained based on the beam model proposed by Park and Gao (2006). By considering corrected material length scale parameter, (Dehrouyeh-Semnani, 2014) showed that the stiffness of microbeam was overestimated by Kong et al. (2009).

Many have researchers employed the aforementioned higher-order elasticity theories to develop microstructure models and investigate size effect in micro scale. Some of works based on modified couple stress theory can be listed as: linear Euler–Bernoulli beam model for vibration analysis by Kong, Zhou, Nie, and Wang (2008), linear Timoshenko beam model

* Tel.: +98 9112268514; fax: +98 1233232701.
E-mail address: A.M.Dehrouye@ut.ac.ir

http://dx.doi.org/10.1016/j.ijengsci.2014.07.008
0020-7225/

by Ma, Gao, and Reddy (2008), linear Kirchhoff plate model for static analysis by Tsiatas (2009), nonlinear Euler–Bernoulli beam model for static bending, free vibration and stability analysis by Xia, Wang, and Yin (2010), nonlinear Timoshenko beam model for static bending and free vibration analysis by Asghari, Kahrobaiyan, and Ahmadian (2010b), linear Mindlin plate model by Ma, Gao, and Reddy (2011), buckling analysis of axially loaded microbeam (Akgöz & Civalek, 2011), linear functionally graded Euler–Bernoulli and Timoshenko beam models for static and free vibration analysis by Asghari, Ahmadian, Kahrobaiyan, and Rahaeifard (2010a) and Asghari, Rahaeifard, Kahrobaiyan, and Ahmadian (2011), static bending analysis of composite laminated beam model by Chen, Li, and Xu (2011), linear functionally graded Euler–Bernoulli, Timoshenko and Reddy beam models for buckling analysis by Nateghi, Salamat-Talab, Rezapour, and Daneshian (2012), pull-in phenomena in micro-cantilever by Baghani (2012), energy release rate of notched beam by Sherafatnia, Kahrobaiyan, and Farrahi (2013), nonlinear geometrically imperfect beam for dynamic analysis by Farokhi, Ghayesh, and Amabili (2013), nonlinear functionally graded piezoelectric beam model by Komijani, Reddy, and Eslami (2014) and Komijani, Reddy, and Ferreira (2013), buckling analysis of microbeam based on higher-order beam theories by Mohammad-Abadi and Daneshmehr (2014), nonlinear pipe conveying fluid for free oscillation and divergence instability analysis by Yang, Ji, Yang, and Fang (2014), buckling analysis of composite laminated microbeam by Mohammad Abadi and Daneshmehr (2014), nonlinear vibrations analysis of functionally graded Mindlin microplates by Ansari, Faghih Shojaei, Mohammadi, Gholami, and Darabi (2014) and yield criterion by Kahrobaiyan, Rahaeifard, and Ahmadian (2014). Moreover, some of works based on modified strain gradient elasticity theory can be listed as: Timoshenko beam model by Wang, Zhao, and Zhou (2010), nonlinear Euler–Bernoulli beam model for static and free vibration analysis by Kahrobaiyan, Asghari, Rahaeifard, and Ahmadian (2011), linear Kirchhoff plate model by Wang, Zhou, Zhao, and Chen (2011) which modified by Ashoori Movassagh and Mahmoodi (2013), buckling analysis of axially loaded microbeam (Akgöz & Civalek, 2011), nonlinear Timoshenko beam model for free vibration and static analysis by Asghari, Kahrobaiyan, Nikfar, and Ahmadian (2012), linear functionally graded Euler–Bernoulli beam model by Kahrobaiyan, Rahaeifard, Tajalli, and Ahmadian (2012), linear functionally graded cylinder model by Sadeghi, Baghani, and Naghdabadi (2012), nonlinear Euler–Bernoulli beam model for forced vibration by Ghayesh, Amabili, and Farokhi (2013), functionally graded curved microbeam by Zhang, He, Liu, Gan, and Shen (2013), trigonometric beam model for buckling analysis by Bekir Akgöz (2014), functionally graded piezoelectric beam model for static and free vibration analysis by Li, Feng, and Cai (2014), cylindrical thin-shell model by Zeighampour and Tadi Beni (2014), thermoelasticity model for Timoshenko micro-beams by Taati, Najafabadi, and Reddy (2014) and yield criterion by Rahaeifard, Ahmadian, and Firoozbakhsh (2014).

Constitutive and Euler–Bernoulli thin plane-strain beam models based on modified couple stress and modified strain gradient elasticity theories are investigated. The experimental bending rigidity of epoxy micro-cantilever reported by Lam et al. (2003) is employed to compare the different non-classical beam models. In addition, static bending behavior of microbeam is studied based on the different non-classical beam models for various boundary conditions i.e. clamped–clamped, clamped-free, clamped–pined and pined–pined.

2. Constitutive microbeam models

Lam et al. (2003) developed modified strain gradient elasticity beam model based on constitutive relations and plane-strain assumption. Modified strain gradient elasticity theory employs three length scale parameter (i.e. ℓ_0, ℓ_1 and ℓ_2) to capture size effect. The governing equation of thin plane-strain microbeam based on modified strain gradient elasticity constitutive relations can be obtained by (see Eqs. (54), (64) and (66) in Lam et al. (2003)):

$$D\frac{\partial^4 w}{\partial x^4} - D^h\frac{\partial^6 w}{\partial x^6} = -q \tag{1}$$

and the associated boundary conditions can be obtained by (see Eqs. (64), (66) and (69) in Lam et al. (2003)):

$$\begin{aligned} &D\frac{\partial^3 w}{\partial x^3} - D^h\frac{\partial^5 w}{\partial x^5} = \overline{Q} \quad \text{or} \quad w = \bar{w} \quad \text{at} \quad x = 0, L \\ &D\frac{\partial^2 w}{\partial x^2} - D^h\frac{\partial^4 w}{\partial x^4} = -\overline{M} \quad \text{or} \quad w' = \bar{w}' \quad \text{at} \quad x = 0, L \\ &D^h\frac{\partial^3 w}{\partial x^3} = 0 \quad \text{or} \quad w'' = \bar{w}'' \quad \text{at} \quad x = 0, L \end{aligned} \tag{2}$$

where D and D^h are bending rigidity and higher-order bending rigidity (see Eq. (67) in Lam et al. (2003)).

$$D = D_0\left[1 + \left(\frac{b_h}{h}\right)^2\right], \quad D^h = D_0\delta^2 \tag{3}$$

where D_0 is classical bending rigidity, b_h and δ are higher-order bending parameters which characterize the thickness dependence of beam bending (see Eq. (68) in Lam et al. (2003)).