International Journal of Engineering Science 85 (2014) 74–89

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Competition between radial expansion and axial propagation in bulging of inflated cylinders with application to aneurysms propagation in arterial wall tissue

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ARTICLE INFO

Article history:
Received 24 January 2014
Received in revised form 19 August 2014
Accepted 28 August 2014
Available online 21 September 2014

Keywords:
Bulging bifurcation
Axial propagation of bulging
Aneurysms

ABSTRACT

In this paper, the (quasi-static) axial propagation of the bulging instability mode in thin-walled cylinders under inflation is analyzed. We present the analytical solution for this particular motion as well as for radial expansion during bulging evolution. For illustration, cylinders that are made of either isotropic incompressible non-linearly elastic materials or doubly fiber reinforced incompressible non-linearly elastic materials are considered. The former model is studied to asses the analytical methodology described in this paper. The latter one is related to soft tissue mechanical response and the study of bulging propagation in these models establishes the connection with the propagation of aneurysms in arterial wall tissue. In particular, we show that bulging instability for these material models under the conditions at hand propagates axially in agreement with the propagation of aneurysms in arterial wall tissue. Furthermore, useful insight has been obtained by examining the admissibility and stability of bulging motion. In particular, it is shown that axial propagation of bulging involves two periods: firstly, pressure remains essentially fixed during the ensuing propagation of the bulging instability mode beyond the onset of bifurcation until a suitable configuration is obtained; secondly, in subsequent motion, for further axial propagation of bulging pressure of inflation must be increased. Hence, the structure in subsequent motion can support higher pressures than the pressure associated with the onset of bulging bifurcation. Configurations can be described by two uniform cylinders joined by a transition zone. On the other hand, radial expansion of bulging is related to a decrease of pressure beyond the onset of bulging. Finite element simulations of some thick-walled cylinders made of both models are also performed. Comparison between numerical and analytical results shows a good agreement qualitatively.

1. Introduction

A variety of authors have focused on (bifurcation) bulging from a membrane tube configuration subject to axial loading and internal pressure (see, Alhayani, Giraldo, Rodriguez, & Merodio, 2013; Merodio & Haughton, 2010; Fu, Pearce, & Liu, 2008; Fu, Rogerson, & Zhang, 2012; Fu & Xie, 2012; Haughton & Merodio, 2009; Haughton & Ogden, 1979; Kyriakides & Chang, 1990, 1991; Rodriguez & Merodio, 2011) and on postbifurcation (Fu et al., 2008). An important application of this

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http://dx.doi.org/10.1016/j.ijengsci.2014.08.008
0020-7225/

analysis is its relation with aneurysms formation and propagation in arterial wall tissue (see, Alhayani et al., 2013; Fu et al., 2008, 2012; Fu & Xie, 2012; Haughton & Merodio, 2009; Haughton & Ogden, 1979; Kyriakides & Chang, 1990, 1991; Merodio & Haughton, 2010; Rodriguez & Merodio, 2011 and references therein). In this work we pay attention to propagation of bulging in cylinders. Previous work with respect to the bulging instability mode has shown that propagation of this instability is a competition between radial expansion and axial propagation (see Baek, Rajagopal, & Humphrey, 2005; Fu et al., 2008, 2012; Kyriakides & Chang, 1991 for instance). Analytical postcritical behavior is difficult to determine and in the literature there exist solutions only for very simple cases, and based on complicated mathematical machinery (Fu et al., 2008, 2012). Here we continue our previous investigations (see Alhayani et al., 2013; Haughton & Merodio, 2009; Merodio & Haughton, 2010; Rodriguez & Merodio, 2011) and focus now on bulging motion with regard to axial propagation of aneurysms. Furthermore, we provide an analytical description of bulging motion that captures axial propagation of bulging.

Bulging, at least in elastomeric tubes, has been shown to be usually the result of a limit load instability (see for instance Kyriakides, 1994; Kyriakides & Chang, 1991). This is not the case always. For instance, if the ends of the tube are open with the axial stretch fixed, localized bulging can occur even if there is not a pressure limit (see Fu et al., 2008). In any case, localized bulging is a bifurcation, which often is initiated at the limit load but for some materials can occur before it. Furthermore, for an infinite or long enough tube, the stability limit is always associated with the onset of a localized bulge. Formation of a localized bulge is a nonlinear (bifurcation) phenomenon and cannot be described by any linear theory. Now, several authors have investigated a bifurcation phenomenon (see for instance Haughton & Merodio, 2009) in which the zero mode is considered to be sinusoidal with axisymmetric deformations while other authors have considered bifurcations in which the zero mode is not sinusoidal (see Fu et al., 2008). In this paper we consider the former analysis in which the onset of bifurcation has previously been related to a maximum pressure. It has been noted that, under these circumstances, bulging bifurcation is likely to occur first before any other instability mode (see e.g. Rodriguez & Merodio, 2011).

With respect to axial propagation of bulging, for closed ended tubes (to be more precise, one of the ends is closed but at the other end air is forced into the tube) made of isotropic elastomers it has been found that a necessary condition for a bulge to propagate is that the pressure-change in volume response of a section of tube constrained to inflate uniformly has an up–down–up behavior (see for instance Kyriakides, 1994). For some materials, the response does not recover to a second stable branch, in which case the bulge keeps growing radially. The propagation of a bulge along a long party balloon has been described in Chater and Hutchinson (1984), among others. It is shown that axial propagation of bulging involves two periods: firstly, pressure remains essentially fixed during the ensuing propagation of the bulging instability mode beyond the onset of bifurcation until a suitable configuration is obtained; secondly, in subsequent motion, for further axial propagation of bulging pressure of inflation must be increased. The propagation pressure (associated with the first period) would correspond to the one obtained using the equal-area rule, i.e. the Maxwell (propagation) pressure. With respect to the opened-ends case, it is not clear how to define the Maxwell line. In Fu et al. (2008), an infinite thin-walled hyperelastic tube that is inflated by an internal pressure, with the axial stretch at infinity maintained at unity was studied. The bifurcation condition and the near-critical behavior was determined analytically. It was shown that there is a bifurcation with zero mode number and that the associated axial variation of near-critical bifurcated configuration admits a locally bulging or necking solution. Bifurcation pressure and propagation pressure do not coincide. In particular, the bifurcation pressure is a little greater than the propagation pressure. Nevertheless, the focus of this analysis was not the fully postbifurcation bulging motion. In this paper we build upon these results and make contact between the analysis given by Fu et al. (2008) and the well established bulging motion analysis for closed ended tubes. In particular we show that for open-ends tubes axial motion of bulging also involves the two periods captured for closed ended tubes. Furthermore, we extend this analysis to fiber reinforced materials that approach artery behavior. To highlight the results, we use pressure-azimuthal stretch responses since they arise naturally from the numerical computations.

Accurate constitutive modeling is needed to evaluate the mechanics of aneurysm formation (see for instance Fu et al., 2012; Gasser et al., 2012; McEniery, Wilkinson, & Avolio, 2007; Schulze-Bauer, Mrth, & Holzapfel, 2003). It is well known that many factors may be involved in this process (Lu, Rateri, Bruemmer, Cassis, & Daugherty, 2012; Watton, Hill, & Heil, 2004). These include, among others, geometry, non-homogeneous material, anisotropy, growth, remodeling, age, etc. Furthermore, there is no physiologically relevant preclinical model of aneurysm formation and propagation. Usually, the decision to remove an aneurysm through a clinical procedure is taken depending on the (size) geometry of the aneurysm as well as its (growth) time evolution. It is clear that development and growth of aneurysms, as well as tumors, cancers, etc, are driven by mechanical aspects of biology. Hence, data associated with pathological histology of arteries are crucial to analyze aneurysms (Gasser et al., 2012). The possibility that aneurysms in arterial wall tissue can be induced by geometrical imperfection and material degradation opens new perspectives. Indeed, from a biological point of view the creation of elastinase, an enzyme that dissolves elastin, is considered to be one of the factors in aneurysm formation. From a mechanics point of view, bulging is possible when the strain stiffening behavior of the collagen fibers that reinforce the arterial wall tissue and give the anisotropic character is not observed in the material mechanical response (Haughton & Merodio, 2009). It then follows that arterial wall tissue constitutive behavior is close to being isotropic rather than to being anisotropic.

In this paper, the bulging bifurcation mode of the cylinder is obtained considering axisymmetric incremental displacements with respect to a deformed configuration. All configurations are in equilibrium. It follows that incremental displacements are null along the hoop direction and do not depend on the azimuthal angle. In Alhayani et al. (2013), a numerical procedure to analyze bifurcation of inflated hyperelastic thick-walled cylinders was developed. The methodology can also follow postbifurcation. It was shown that in the competition between radial expansion and axial propagation of the