International Journal of Engineering Science 85 (2014) 90–104

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Thermo-mechanical buckling behavior of functionally graded microbeams embedded in elastic medium

Bekir Akgöz, Ömer Civalek *

Akdeniz University, Civil Engineering Department, Division of Mechanics, Antalya, Turkey

ARTICLE INFO

Article history:
Received 5 August 2014
Received in revised form 23 August 2014
Accepted 28 August 2014
Available online 21 September 2014

Keywords:
Thermal effect
Modified couple stress theory
Buckling
Functionally graded materials
Microbeam

ABSTRACT

Thermo-mechanical size-dependent buckling analysis of embedded functionally graded (FG) microbeams is performed based on sinusoidal shear deformation beam and modified couple stress theories. It is assumed that material properties vary smoothly and continuously throughout the thickness. Winkler elastic foundation model is used to simulate the interaction between FG microbeam and elastic medium. The governing equations and corresponding boundary conditions are obtained with the aid of minimum total potential energy principle. The buckling characteristics of simply supported embedded FG microbeams in thermal environment are investigated. The obtained results are compared with the results of simple beam theory with no shear deformation effects and classical theory. Influences of thickness-to-material length scale parameter ratio, material property gradient index, slenderness ratio, temperature change and Winkler parameter on critical buckling loads of embedded FG microbeams are discussed in detail.

1. Introduction

Functionally graded materials (FGMs) can be defined as a new improved kind of composite materials. Unlike in layered composites, material properties vary continuously and smoothly throughout the certain dimension in FGMs. In traditional laminated composites, the interconnected each layer has different material properties. Unfortunately, due to sudden change in material properties, high shear stress problem may occur at the interface of two adjacent layers. It can be emphasized that FGMs may be a good solution for these undesirable stress concentrations. Consequently, structures made of FGMs have a wide range of applications in many industries such as aerospace, biomedicine, mechanical, nuclear, electronics and optics due to their novel thermo-mechanical properties. There are many studies in the literature on investigation mechanical characteristic of functionally graded (FG) structures with various solution methods (Aydogdu & Taskin, 2007; Ferreira, Batra, Roque, Qian, & Jorge, 2006; Ferreira, Batra, Roque, Qian, & Martins, 2005; Neves et al., 2013; Neves et al., 2012; Qian, Batra, & Chen, 2004; Ying, Lü, & Chen, 2008; Zenkour, 2005).

Microbeams are one of the essential structures in micro- and nano-electro mechanical systems (MEMS and NEMS) such as micro-resonators (Zook et al., 1992), Atomic Force Microscopes (Torii, Sasaki, Hane, & Okuma, 1994), micro-switches (Acquaviva et al., 2009) and micro-actuators (Hung & Senturia, 1999). Some experimental observations demonstrated that size effect plays an important role on mechanical characteristics of small-sized structures (Fleck, Muller, Ashby, & Hutchinson, 1994; Lam, Yang, Chong, Wang, & Tong, 2003; Stolken & Evans, 1998). For instance, it is observed by Lam

---

* Corresponding author. Tel.: +90 242 3106319.
E-mail address: civalek@yahoo.com (Ö. Civalek).

http://dx.doi.org/10.1016/j.ijengsci.2014.08.011
0020-7225/

et al. (2003) in micro bending test of epoxy beams that the normalized bending rigidity increases about 2.4 times as the thickness of the beam reduces from 115 to 20 μm. Classical continuum mechanics is not sufficient to predict the size dependency of micro- and nano-sized structures in the absence of any material length scale parameters. Therefore, several nonclassical continuum theories, in which there is at least one additional material length scale parameter, have been developed to determine the mechanical characteristics of small-sized structures, such as couple stress theory (Koiter, 1964; Mindlin & Tiersten, 1962; Toupin, 1964), micropolar theory (Eringen, 1967), nonlocal elasticity theory (Eringen, 1972, 1983) and strain gradient theories (Aifantis, 1999; Fleck & Hutchinson, 1993, 2001; Lam et al., 2003; Vardoulakis & Sulem, 1995).

The modified couple stress theory (MCST) was elaborated by Yang, Chong, Lam, and Tong (2002) in which strain energy density includes symmetric rotation gradient tensors besides symmetric strain tensor. For linear elastic isotropic materials, the formulations and governing equations contain only one additional material length scale parameter besides two classical ones. A number of studies have been performed based on this theory in order to investigate mechanical responses of homogeneous microbeams (Ghayesh, Amabili, & Farokhi, 2013a, 2013b; Ghayesh, Farokhi, & Amabili, 2013; Kong, Zhou, Nie, & Wang, 2008; Ma, Gao, & Reddy, 2008; Park & Gao, 2006), microplates (Akgöz & Civalek, 2012a, 2013a; Asghari, 2012; Jomehzadeh, Noori, & Saidi, 2011; Tsiatas, 2009; Yin, Qian, Wang, & Xia, 2010).

Another type of higher-order continuum theories is modified strain gradient theory (MSGT) proposed by Lam et al. (2003). This theory contains a new higher-order equilibrium relation for moments of couples in addition to well-known classical equilibrium relations for forces and moments of forces and only one additional material length scale parameter. Unlike MCST, this theory includes two additional material length scale parameters corresponding to dilatation gradient vector and deviatoric stretch gradient tensor in addition to symmetric rotation gradient tensor. This theory has been performed to investigate mechanical behaviors of homogeneous microbars (Akgöz & Civalek, 2014a; Güven, 2014; Kahrobaiyan, Tajalli, Movahhedy, Akbari, & Ahmadian, 2011, 2013) and microbeams (Akgöz & Civalek, 2011, 2012b; Asghari, Kahrobaiyan, Nikfar, & Ahmadian, 2012; Ghayesh & Farokhi et al., 2013; Kong, Zhou, Nie, & Wang, 2009; Wang, Zhao, & Zhou, 2010).

Nowadays, FG microstructures are extensively used in MEMS and NEMS (Fu, Du, Huang, Zhang, & Hu, 2004; Rezazadeh, Tahmasebi, & Zubtsov, 2006; Witvrouw & Mehta, 2005) due to the rapid developments in nanotechnology. Several studies have been performed to determine mechanical characteristics of FG microbars and microbeams. For instance, Akgöz and Civalek (2013b) investigated longitudinal free vibration analysis of axially functionally graded microbars based on MSGT in conjunctions with Rayleigh–Ritz solution method. Sadeghi, Baghani, and Naghdabadi (2012) presented a study about the effect of material length scale parameters on analysis of strain gradient FG micro-cylinders. Static bending and free vibration analysis of simply supported FG microbeams based on Bernoulli–Euler beam model and MSGT was investigated by Kahrobaiyan, Rahaeifard, Tajalli, and Ahmadian (2012). Also, buckling analysis of strain gradient FG microbeams for different boundary conditions on the basis of Bernoulli–Euler beam theory was performed by Akgöz and Civalek (2013c). A size-dependent strain gradient Timoshenko beam model for nonhomogeneous microbeams was presented by Ansari, Gholami, and Sahmani (2011).

The abovementioned studies have been performed on the basis of Bernoulli–Euler (EBT) and Timoshenko beam theories (TBT). Effects of shear deformation are neglected in assumption EBT. This simple theory can be successfully used in analysis of slender beams with a large aspect ratio. However, influences of shear deformation may become more considerable for moderately thick beams. TBT, known as first-order shear deformation beam theory, assumes that transverse shear stress and strain are uniform along the thickness of the beam. An appropriate shear correction factor is needed in the formulations due to the zero transverse shear stress and strain condition at the upper and lower surfaces of the beam. Higher-order shear deformation beam theories including parabolic (third-order) beam theory (PBT) (Levinson, 1981; Reddy, 1984), trigonometric (sinusoidal) beam theory (SBT) (Touratier, 1991), hyperbolic beam theory (Soldatos, 1992), exponential beam theory (Karama, Afaq, & Mistou, 2003) and a general exponential beam theory (Aydogdu, 2009) have been proposed to satisfy the zero transverse shear stress and strain condition at the top and bottom surfaces of the beam without any shear correction factors. There are several works on investigation mechanical responses of such structures on the basis of modified couple stress and strain gradient theories (Akgöz & Civalek, 2013d, 2014b, 2014c; Lei, He, Zhang, Gan, & Zeng, 2013; Nateghi, Salamat-talab, Rezapour, & Daneshian, 2012; Reddy, 2011; Salamat-talab, Nateghi, & Torabi, 2012; Simsek & Reddy, 2013a, 2013b).

On the other hand, temperature changes may become very important depending on material and geometric properties of structures. Several studies have been performed to investigate the thermal effect on mechanical responses of beams and plates. Thermal buckling and vibration analysis of FG beams was performed based on an improved third-order shear deformation beam theory by Wattanasakulpong, Prusty, and Kelly (2011). Thermo-mechanical buckling and nonlinear free vibration behaviors of embedded FG beams on nonlinear elastic foundation were investigated on the basis of Euler–Bernoulli beam theory in conjunction with von Karman strain displacement relation by Fallah and Aghdam (2012). Recently, Yaghoobi and Fereidoon (2014) proposed a refined *n*th-order shear deformation theory for investigation the mechanical and thermal buckling responses of FG plates resting on elastic foundation. Also, thermo-mechanical shear buckling analysis of orthotropic single-layered graphene sheets based on nonlocal elasticity theory was performed by Mohammadi, Farajpour, Moradi, and Ghayour (2014). Ke, Wang, and Wang (2011) and Nateghi and Salamat-talab (2013) presented a study on free vibration and buckling characteristics of the homogeneous and functionally graded microbeams in thermal environment based on modified couple stress theory and Timoshenko beam theory, respectively. Microstructure-dependent buckling response of functionally graded third-order shear deformable microbeams in thermal environment was investigated based on modified strain gradient elasticity theory by Sahmani and Ansari (2013). More recently, nonlinear vibration and post-