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Analytical solutions of 3D anisotropic magneto-electro-elastic bi-materials under extended non-uniform dislocations and tractions over a circular area

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ABSTRACT

In this paper, we derive the analytical solutions in a three-dimensional magneto-electro-elastic (MEE) bi-material under extended dislocations and tractions generally distributed over a horizontal circular area by virtue of the extended Stroh formalism and Fourier transformation. Explicit analytic results of the Hankel transform integrals of different orders and three kernel integrals are obtained so that the final solutions in the physical domain can be expressed in simple line-integral form. Some existing solutions in MEE materials can be reduced directly from the general solutions presented in this paper. The effect of different non-uniform loadings (dislocations and tractions) on the induced fields in MEE bi-material is investigated. Our analytical solutions can serve as benchmarks for future numerical analysis of various problems in MEE bi-materials.

1. Introduction

Since $BaTiO_3$–$CoFe_2O_4$ material was first fabricated in 1974 (Van Run, Terrell, & Scholing, 1974), increasing attention has been paid to the studies of the coupling features in three-dimensional (3D) magneto-electro-elastic (MEE) materials. Although materials possessing magneto-electric coupling can be found in nature, the coupling effect in them is very small. On the other hand, composites made of piezoelectric and piezomagnetic phases could produce a large magneto-electric coupling effect (Kuo & Peng, 2013). So far, various modeling methods have been proposed to study the properties and behaviors of MEE composite materials (Kuo & Peng, 2013; Aboudi, Zheng, & Jin, 2014; Kuo, 2011). Wang and Shen (2002) derived the Green's functions in the transversely isotropic (TI) infinite MEE material by applying the potential function approach. Ding and Jiang (2003) derived the fundamental solutions for the TI MEE half-space in terms of elementary functions by the trial-and-error method. Solutions of the MEE half space under circular loadings on the surface were presented by Wang, Pan, Sangghaleh, Wang, and Han (2012).

While experimental and numerical simulations are available to provide specific parametric studies of MEE materials, suitable analytical solutions are needed to analyze the general and global behaviors of MEE materials, especially for the MEE bi-materials whose interface could substantially influence the coupling behaviors of the materials. Actually, Pan (2002) derived the Green's functions for the anisotropic MEE 3D bi-material by the extended Stroh formalism and Fourier transformation

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Nomenclature

c_{iJKl}	extended stiffness matrix elements
u_K	extended displacement components
f_J	extended body forces
d_J	extended dislocations
T_J	extended tractions
σ_{iJ}	extended stresses
δ	delta function
x_i	rectangular coordinates in physical domain
ρ, ψ	polar coordinates in physical domain, ψ in Section 5.1 also denotes magnetic potential
k_i	rectangular coordinates in Fourier transformed domain
η, θ	polar coordinates in Fourier transformed domain
φ	electric potential

with the solutions being expressed in terms of the simple line integrals. Ding, Jiang, Hou, and Chen (2005) obtained the Green's functions of two-phase TI MEE media for both 2D and 3D cases via the harmonic displacement functions. Zhao, Shang, and Pan (2013) derived the analytical solutions of the MEE bi-material system under simple internal dislocations and tractions distributed over a horizontal circular area.

We also point out that various defects may exist in MEE materials and that such defects may potentially influence the material behaviors. Particularly, lattice mismatch and thermal mismatch between adjacent layers would induce strain fields into the materials, which could be partially relaxed via the formation of misfit/threading dislocations (Chu, Wang, Zhou, & Beyerlein, 2011). Furthermore, the dislocation or displacement-discontinuity method is also one of the common computational methods in the study of fracture problems (Fan, Zhao, Zhao, & Pan, 2012; Hao & Liu, 2006; Li & Lee, 2010; Zhao, Zhao, Pan, & Fan, 2014). However, as for dislocation problems in 3D MEE materials, only very little work has been done (Chu, Wang, Zhao, & Pan, 2013; Han & Pan, 2013).

In this paper, we present analytical solutions of generally non-uniform loadings in 3D MEE bi-materials. It is an extension of our recent work in Zhao et al. (2013) (hereafter is called Paper I) to the general dislocations and tractions loading cases. Thus, the solutions presented in this paper contain those in Paper I and also those in Wang et al. (2012) as their special cases. It should also be pointed out that the obtained analytical solutions can serve as special kernel functions in various boundary integral equation methods as well as benchmarks to the numerical methods in 3D MEE material analyzes.

This paper is organized as follows: In Section 2, the mathematical model similar to the one in Paper I is briefly reviewed. In Section 3, the general solutions in the Fourier-transformed domain as well as the double integration representations of the solutions in the physical domain are presented. In Section 4, final physical-domain solutions of generally non-uniform loadings are derived and expressed in simple line integration form. In Section 5, numerical examples are presented to show the effect of different non-uniform loadings (dislocations and tractions) on the induced field in the MEE bi-materials. Conclusion is drawn in Section 6.

2. Mathematical model of the problem

As an extension of Paper I, the problem description is briefly presented for easy reference. A bi-material system of linear anisotropic MEE is considered (Fig. 1). The upper ($x_3 > 0$) and lower ($x_3 < 0$) half spaces are assigned as Materials 1 and 2,

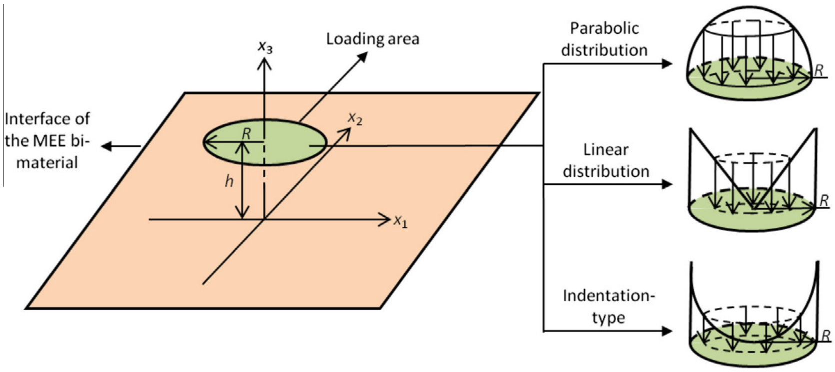

Fig. 1. An MEE bi-material under a general internal loading (traction or dislocation) over a horizontal circular area of radius R, with three types of loadings being illustrated.