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Propagation of surface SH waves on a half space covered by a nonlinear thin layer

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ABSTRACT

The self modulation of surface shear horizontal (SH) waves (Love waves) propagating on a nonlinear half space covered by a nonlinear thin layer is examined. First a nonlinear thin layer approximation is derived by assuming that constituent materials are nonlinear,homogeneous, isotropic and compressible hyper-elastic. Then employing this approximation, a two medium problem is reduced to one for a nonlinear half space with a modified nonlinear boundary condition on the top surface. This new problem is analyzed by the method of multiple scales, and a nonlinear Schrödinger (NLS) equation is derived which describes the self modulation of the waves asymptotically. Then the results of the nonlinear thin layer approximation are compared with the results of the linear thin layer approximation and with the finite layer results obtained in a previous work. It has been observed that even in moderately low wave numbers, the propagation is affected considerably by the nonlinear material parameter of the layer.

1. Introduction

Elastic waves propagating in wave guides are dispersive, i.e. the phase velocities of waves depends on the wave number. Linear dispersive elastic waves have been studied extensively, because of their important applications in geophysics, non-destructive testing of materials, electronic signal processing devices, etc. (see, e.g. Achenbach, 1973; Ewing, Jardetsky, & Press, 1957; Eringen & Suhubi, 1975; Farnell, 1978; Maugin, 1983; Miklowitz, 1978 and references there in). Recently, the effect of constitutional nonlinearities on the propagation of these waves had been the subject of many investigations. In most of these works asymptotic perturbation methods, previously used in fluid mechanics, plasma physics, lattice dynamics etc., to investigate the propagation of weakly nonlinear waves in these fields, (see e.g. Ablowitz & Clarkson, 1991; Dodd, Eilbeck, Gibbon, & Morris, 1982; Jeffrey & Kawahara, 1981; Johnson, 1997; Whitham, 1974), have been employed. In the small but finite amplitude limit balancing the nonlinearity and dispersion in the analysis several different types of nonlinear evaluation equations such as Korteweg-de Vries (K-dV), modified K-dV, nonlinear Schrodinger (NLS), Boussinesq (BE), modified BE equations, etc. have been derived to describe the wave propagation asymptotically. Then several aspects of problems, such as nonlinear stability of modulated waves, steady state solutions, the occurrence of various types of solitary waves, etc. were discussed on the basis of these equations (see e.g. Ahmetolan & Teymur, 2007; Bataille & Lund, 1982; Demiray, 2008; Destrade, Gilchrist, & Saccomandi, 2010a; Destrade, Gilchrist, & Ogden, 2010b; Destrade, Goriely, & Saccomandi, 2011; Destrade & Saccomandi, 2005; Destrade & Saccomandi, 2006; Fu, 1996; Kalyanasundaram, 1981; Maugin & Hadouaj,

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1991; Mayer, Parker, & Maradudin, 1992; Maugin, Hadouaj, & Malomed, 1992; Porubov & Samsonov, 1995; Soerensen, Christiansen, & Lomdahl, 1984; Teymur, 1988; Teymur, 1996; Teymur, 2007). For an extensive review of most of these works we refer to Parker and Maugin (1988), Maugin (1994), Parker (1994), Samsonov (1994), Mayer (1995), Norris (1998), Porubov (2003). Among them the investigations of nonlinear surface shear horizontal (SH) waves (Love waves) propagating in an elastic half space covered by an elastic layer of uniform thickness with a different material properties, occupies an important place. Some of these works will be reviewed here in detail as the work presented here is related with this subject.

As far as we have traced, nonlinear Love waves were first examined by Kalyanasundaram (1981). In Kalyanasundaram (1981) the propagation of quasi-monochromatic waves are studied by employing the multiple scale method. A semi-linear partial differential equation governing the slow variation in complex amplitude has been derived. This equation is reduced to a pair of semi-linear hyperbolic equations for the real amplitude and the phase of the quasi-monochromatic waves. The solutions of these equations show that the amplitude of the waves remains constant along the characteristics and the wave is subject only to a phase shift. In the analysis the balance between nonlinearity and dispersion does not take place, therefore a localized nonlinear wave of permanent form, a solitary wave, is not present as asymptotic solution form. Later by Bataille and Lund (1982), a model equation is derived via an intuitive approach guided by physical arguments to take into account the dispersive nature of Love waves and the nonlinearity. Here the layer is considered as linear thin elastic layer and a linear dispersive equation is written to describe the equation of this layer.The equation of motion of the half space is approximated as an elastic medium with a cubic nonlinearity. Then combining these equations together, i.e. balancing the nonlinearity of the half space by the dispersion produced by the thin elastic layer, a modified BE equation with a higher order nonlinearity than the usual BE equation, is obtained. It is then shown that this equation has an exact solitary wave and an approximate modulated solitary wave (an envelope solitary wave) solution which provides mechanisms for localized energy propagation along the surface of the layered medium under consideration. Because of the thin linear elastic layer approximation, these observations are valid on the first branch of the linear dispersion relation of the classical Love waves as $kh \to 0$ where k is the wave number and h is the thickness of the layer. It is known that, in an hyper-elastic material nonlinearity causes shear waves to derive longitudinal components of the displacement field (see e.g. Eringen & Suhubi, 1974). Therefore, as it is discussed by Carroll (1967), only in a special hyper-elastic material, it is possible to create an anti-plane motion (a generalized shear motion) without application of external forces acting in the plane perpendicular to the displacement vector. This fact has not been taken into account in the works done by Kalyanasundaram (1981), Bataille and Lund (1982). Later in Teymur (1988), the propagation of nonlinear Love waves in a half space covered by a layer of uniform finite thickness having different mechanical properties, is considered. The materials of the layer and the half space are both assumed to be homogeneous, isotropic and compressible hyper-elastic. Further, it is also assumed that the materials are restricted so that the shear horizontal surface waves can be maintained without applying external forces in the plane perpendicular to the displacement vector. Then utilizing the derivative expansion method and balancing the nonlinearity and dispersion in the asymptotic analysis, it is shown that the nonlinear self modulation of Love waves is governed by an NLS equation. Since in the analysis the thickness h of the layer is kept finite, i.e. there is no restriction on h and also on k to be small, the coefficients of the NLS equation are valid on all branches of the linear dispersion relation of Love waves for any wave number. These coefficients also depend on linear and nonlinear material constants of the constituent materials. From the numerical evaluation of these coefficients for various material parameters it has been observed that the stability of modulated waves, the existence of envelope (bright) and dark solitary waves depend strongly on the nonlinear properties of the layered media as well as the wave number. Later in Maugin and Hadouaj (1991), the problem is examined when the nonlinear substrate covered by a linear elastic interface of mathematically vanishing thickness. The equation of motion of the substrate approximated by a nonlinear equation having a third order nonlinearity as in Teymur (1988). The analysis is simplified by use of a linear thin film approximation which converts a two medium problem by one for a half space with a modified boundary condition on the top surface. The problem is then investigated asymptotically by the Whitham–Newell technique and also by the Hayes method. For nonlinear dispersive almost monochromatic waves an NLS equation is obtained at the interface. Then the existence of dark and bright acoustic solitary waves for some real materials are discussed. The propagation of large amplitude Love waves in a layered half-space made of different pre-stressed compressible neo-Hookean materials (restricted Hadamard materials) is examined by Ferreira and Boulanger (2008) and an exact solution of the problem is given. Later, the anti-plane shear motions coupled with an in-plane motion for a Hadamard materials are considered by Pucci and Saccomandi (2013a, 2013b). The pure anti-plane motion may be sustained in a Hadamard material in the absence of body forces. When the constitutive parameter β is small, a perturbation analysis is developed using this small parameter. Then this approach is also applied to examine the propagation of finite amplitude Love waves in a layered half space made of Hadamard materials. And the solutions exhibiting a secondary in-plane motion caused by a principal anti-plane motion are given.

In the present work, we extend the investigation of Love waves to the case in which the thin layer is made of a nonlinear material so that the modified boundary condition written on the top of the surface includes the nonlinear material parameters of the layer. The nonlinear thin layer model is obtained by employing the governing equations derived in Teymur (1988) for the propagation of shear horizontal surface waves. Then, employing this nonlinear thin layer approximation, the problem is reduced, as in Maugin and Hadouaj (1991), to a problem for a nonlinear half-space with a modified boundary condition on the top surface that includes the nonlinear material parameter of the layer. The propagation of nonlinear surface waves on this structure is investigated by a multiple scale perturbation method (see e.g. Jeffrey & Kawahara, 1981) and an NLS equation is derived describing the self modulation of these waves as in the previous works of Teymur (1988) and Maugin and Hadouaj (1991). We have to note that in Teymur (1988) an NLS is derived for a finite nonlinear layer