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Rayleigh waves with impedance boundary conditions in incompressible anisotropic half-spaces

Pham Chi Vinh [a,*], Trinh Thi Thanh Hue [b]

[a] Faculty of Mathematics, Mechanics and Informatics, Hanoi University of Science, 334, Nguyen Trai Str., Thanh Xuan, Hanoi, Viet Nam
[b] Faculty of Civil and Industrial Construction, National University of Civil Engineering, 55 Giai Phong Str., Hanoi, Viet Nam

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ABSTRACT

In this paper, the propagation of Rayleigh waves in an incompressible elastic half-space with impedance boundary conditions is investigated. The half-space is assumed to be orthotropic and monoclinic with the symmetry plane $x_3 = 0$. The main aim of the paper is to derive explicit secular equations of the wave. For the orthotropic case, the secular equation is obtained by employing the traditional approach. It is an irrational equation. For the monoclinic case, the method of polarization vector is used for deriving the secular equation. This is an algebraic equation of eighth-order. When the impedance parameters vanish, the equations obtained coincide with the corresponding secular equations of Rayleigh waves with traction-free boundary conditions.

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1. Introduction

Elastic surface waves, discovered by Rayleigh (1885) more than 120 years ago for compressible isotropic elastic solids, have been studied extensively and exploited in a wide range of applications in seismology, acoustics, geophysics, telecommunications industry and materials science, for example. For Rayleigh waves their explicit secular equation are important in practical applications. They can be used for solving the direct (forward) problems: evaluating the dependence of the wave velocity on material parameters, especially for solving the inverse problems: to determine material parameters from measured values of wave velocity. Therefore, explicit secular equations are always the main purpose for any investigation of Rayleigh waves.

In the context of Rayleigh waves, it is almost always assumed that the half-spaces are free of traction. As mentioned in Godoy, Durn, and Ndlec (2012), in many fields of physics such as acoustics and electromagnetism, it is common to use impedance boundary conditions, that is, when a linear combination of the unknown function and their derivatives is prescribed on the boundary. See, for examples, Antipov (2002), Zakharov (2006), Yla-Oijala and Jarvenppa (2006), Mathews and Jeans (2007), Castro and Kapanadze (2008) and Qin and Colton (2012), for the acoustics case and Senior (1960), Asghar and Zahid (1986), Stupfel and Poget (2011) and Hiptmair, Lopez-Fernandez, and Paganini (2014) for the electromagnetism one, and the references therein. In the other hand, when studying the propagation of Rayleigh waves in a half-space coated by a thin layer, the researchers often replace the effect of the thin layer on the half-space by the effective boundary conditions on the surface of the half-space, see, for examples, Achenbach and Keshava (1967), Tiersten (1969), Bovik (1996), Steigmann and Ogden (2007), Vinh and Khanh Linh (2012, 2013), Vinh and Anh (2014a, 2014b) and Vinh, Anh, and Thanh (2014). These

* Corresponding author. Tel.: +84 4 35532164; fax: +84 4 38588817.
E-mail address: pcvinh@vnu.edu.vn (P.C. Vinh).

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conditions lead to the impedance boundary conditions on the surface. The Rayleigh is then considered as a surface wave that propagates in a half-space without coating whose surface is not traction-free but is subjected the impedance boundary conditions. As addressed in Makarov, Chilla, and Frohlich (1995) and Niklasson, Datta, and Dunn (2000), a thin layer attached to a half-space is a model finding a broad range of applications in modern technology. Rayleigh waves with impedance boundary conditions are therefore needed to be investigated. However, very few investigations on Rayleigh waves with impedance boundary conditions have been done. Malischewsky (1987) considered the propagation of Rayleigh waves with Tiersten's impedance boundary conditions and provided a secular equation. Recently, Godoy et al. (2012) investigated the existence and uniqueness of Rayleigh waves with impedance boundary conditions which are a special case of Tiersten's impedance boundary conditions. In Godoy et al. (2012) and Malischewsky (1987), the half-space is assumed to be isotropic. Note that the Tiersten impedance boundary conditions are not accurate ones.

The main purpose of this paper is to study the propagation of Rayleigh waves with Tiersten's impedance boundary conditions (Malischewsky, 1987) in anisotropic incompressible elastic half-spaces. Two cases of anisotropy are considered: orthotropic materials and monoclinic ones with the symmetry plane $x_3 = 0$. For the orthotropic case, the secular equation is obtained by employing the traditional techniques. It is an irrational equation. For the monoclinic case, for obtaining the secular equation we use the method of polarization vector. The secular equation obtained is an algebraic equation of eighth-order. When the impedance parameters vanish, the obtained equations coincide with the corresponding secular equation of Rayleigh waves with traction-free boundary conditions.

2. Orthotropic half-spaces

Consider an elastic half-space which occupies the domain $x_2 \geqslant 0$. We are interested in the plane strain such that:

$$u_i = u_i(x_1, x_2, t), \quad i = 1,2, \; u_3 \equiv 0 \tag{1}$$

where t is the time. Suppose that the half-space is made of incompressible orthotropic elastic material, then the strain–stress relations are (Nair & Sotiropoulos, 1997):

$$\begin{cases} \sigma_{11} + p = c_{11}u_{1,1} + c_{12}u_{2,2} \\ \sigma_{22} + p = c_{12}u_{1,1} + c_{22}u_{2,2} \\ \sigma_{12} = c_{66}(u_{1,2} + u_{2,1}) \end{cases} \tag{2}$$

where σ_{ij} and c_{ij} are respectively the stresses and the material constants, $p = p(x_1, x_2, t)$ is the hydrostatic pressure associated with the incompressibility constraint, commas indicate differentiation with respect to spatial variables x_k. The elastic constants $c_{11},\; c_{22},\; c_{12},\; c_{66}$ satisfy the inequalities:

$$c_{ii} > 0, \quad i = 1,2,6, \quad c_{11} + c_{22} - 2c_{12} > 0 \tag{3}$$

which are necessary and sufficient conditions for the strain energy of the material to be positive semi-definite. For an incompressible material, we have:

$$u_{1,1} + u_{2,2} = 0 \tag{4}$$

from which we deduce the existence of a scalar function, denoted $\psi(x_1, x_2, t)$, such that:

$$u_1 = \psi_{,2}, \quad u_2 = -\psi_{,1} \tag{5}$$

In the absence of body forces, equations of motion are:

$$\begin{cases} \sigma_{11,1} + \sigma_{12,2} = \rho\ddot{u}_1 \\ \sigma_{12,1} + \sigma_{22,2} = \rho\ddot{u}_2 \end{cases} \tag{6}$$

where ρ is the mass density, a superposed dot signifies differentiation with respect to t. Introducing Eqs. (2) and (5) into Eq. (6) and eliminating p from the resulting equations lead to an equation for ψ, namely:

$$c_{66}\psi_{,1111} + (c_{11} - 2c_{12} + c_{22} - 2c_{66})\psi_{,1122} + c_{66}\psi_{,2222} = \rho\left(\ddot{\psi}_{,11} + \ddot{\psi}_{,22}\right) \tag{7}$$

Consider the propagation of a Rayleigh wave, traveling with velocity $c(> 0)$ and wave number $k(> 0)$ in the x_1-direction and decaying in the x_2-direction, i. e.:

$$u_i \to 0 \quad (i = 1,2) \quad \text{as} \quad x_2 \to +\infty \tag{8}$$

Suppose that the surface $x_2 = 0$ is subjected to impedance boundary conditions such that (Godoy et al., 2012; Malischewsky, 1987):

$$\sigma_{12} + \omega Z_1 u_1 = 0, \quad \sigma_{22} + \omega Z_2 u_2 = 0 \quad \text{at} \quad x_2 = 0 \tag{9}$$

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