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A multilayer film coating with slightly curved boundary

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A B S T R A C T

Surface irregularity is often a source of stress concentrations. They can be reduced by proper matching of dissimilar surface layers. In this connection, a 2-D model of an isotropic multilayer film-substrate composite with rough surface is considered, in order to examine an influence of physical and geometrical properties of the components on the stress state of a boundary. Using the complex variable representations and boundary perturbation technique, the problem is reduced to the set of Fredholm integral equations of the second kind for an arbitrary shape of the boundary. The solution is given in terms of Fourier series for the periodic shape. In the case of a bilayer film coating, numerical results obtained with the use of the first-order approximation demonstrate effects of the boundary shape, thickness and elastic properties of the layers and the substrate on the stress state of a rough surface.

1. Introduction

In recent years, much attention has been paid to analysis of multilayer thin film structures. In many applications, the use of multilayer coatings leads to an improvement of mechanical, optical, electrical and magnetic properties of devices. However, the high device quality can only be maintained if the defects in films are kept to minimum.

One of these defects is the surface roughness which can be caused by a number of natural phenomena (Freund & Suresh, 2003; Medina & Hinderliter, 2014; Pronina, 2013): heat, light, radioactive emissions, chemicals, mechanical stress etc. Furthermore, during the film deposition and subsequent thermal processing, the film surface frequently becomes unstable due to diffusion and evolves into an undulating profile with cusp-like valleys. Some of the works containing analytical solutions of corresponding 2-D problems or experimental investigations one can find in review of Gao and Nix (1999). Analyzing a sinusoidal and a single wave perturbation of a flat surface of a stressed solid in 2-D and 3-D problems of elasticity, Gao (1991) has shown that even a slightly undulate surface can generate significant stress concentration that may cause fracture before the bulk stress reaches a critical level (Gao, 1991a, 1991b). Similar effect of a non-planar bimaterial interface in 2-D problem on the interface fracture was reported by Evans and Hutchinson (1989).

Stress distribution in a subsurface and stress concentration caused by a surface irregularity are principally studied using experimental technique and finite element method (see, for example Gunnars, Wikman, & Hogmark, 1997; Kainuma, Jeong, & Ahn, 2014; Medina & Hinderliter, 2012, 2013). Most of analytical solutions considering the surface undulation were intended for the investigation of morphological instability of a surface (Freund & Jonsdottir, 1993; Gao, 1991; Kim & Vlassak, 2007; Li, Wang, & Fan, 2008; To, 2012). An exception is a study of stress concentration at slightly sinusoidal surface (local cavity or periodic undulation in one and two directions) by Gao (1991a), randomly slightly damaged surface of a solid by Medina and Hinderliter (2014). Besides of surface morphology instability of elastically anisotropic solids, To (2012) has

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also examined stress concentration for the anisotropic solid with a free surface slightly undulated towards one and two directions. All of these works are limited to constructing only first order accurate solution of perturbation technique and considering the sinusoidal undulation of a boundary surface. Such a simplest shape of a surface are not suitable for describing a wide range of surface profiles observed in experiments (Jesson, Pennycook, & Baribeau, 1993; Yao, Andersson, & Dunlop, 1994). Apart from Kim and Vlassak (2007) investigated unstable sinusoidal surface of multilayered thin films structure, the rest mentioned works consider eider a single material system or one-layer film bonded to a substrate.

The motivation of the present work is based on the fact that thin film coating may consist of a number of layers and have a different surface profiles. The foundation for solving corresponding problem and analyzing the influence of a slight surface non-flatness on the stress distribution in the multilayered film coating is a universal boundary perturbation method applied to different 2-D problems of elasticity (Grekov & Makarov, 2004; Grekov, 2004, 2011; Grekov & Morozov, 2009; Grekov, Morozov, & Yazovskaya, 2014; Vikulina, Grekov, & Kostyrko, 2010). In each case, an algorithm for obtaining any-order approximation is constructed. Following this method, the solution for a single local distortion of a surface (Grekov & Makarov, 2004) and an interface (Grekov, 2004, 2011) and for a curvilinear interfacial crack (Grekov, 2011) in bimaterial was expressed in quadratures. Effects of these defects were analyzed in the framework of the first-order approximation. In the case of a single-layer film coating with a slightly curved surface (Vikulina et al., 2010) or interface (Grekov & Kostyrko, 2013) and for a curvilinear crack near an interface in bimaterial (Grekov & Morozov, 2009), the boundary perturbation method was used with the superposition technique that leads to the solution of the Fredholm integral equation of the second kind. For every order accurate solution in perturbation, the right hand side of this integral equation depends of all previews approximations. The corresponding integral equation for the first-order approximation was solved by Vikulina et al. (2010) and Grekov and Kostyrko (2013) in terms of Fourier series when curvilinear surface/interface of the single-layer film coating is described by an arbitrary periodic function. Note that the boundary perturbation method has been also used to solve the problem of nanoscale nearly circular holes, and the procedure involved a hypersingular integral equation for any-order approximation (Grekov et al., 2014).

In the present work, we generalized the approach used by Vikulina et al. (2010) to the multilayered film coating in order to examine the possibilities of reduced stress concentration by proper matching of dissimilar layers at the substrate. Using the complex variable representations, specific superposition method and boundary perturbation technique, we derive the system of integral equations assumed that the slightly curved surface of the coating has an arbitrary shape. Following the algorithm described in the paper, one can numerically obtain any-order approximate solution of the problem. Numerical results and analysis of the non-uniform stress state at the rough surface of two-layered coating is represented in the Section 8. For this purpose, we use the special periodic function to describe a wide range of surface configurations: from wavy surface considered by Kim and Vlassak (2007) to periodically distributed local surface defects such as grooves and asperities.

2. Formulation of the problem

Consider the 2D problem for a thin film coating of the total thickness $h_f = \sum_{r=1}^{N} h_r$, which consists of N dissimilar layers and is deposited on a substrate with Poisson's ratio ν_{N+1} and shear modulus μ_{N+1} (Fig. 1). The layer of thickness h_r has Poisson's ratio ν_r and shear modulus μ_r.

The substrate is modeled as an elastic half-plane of complex variable $z = x_1 + ix_2$

$$\Omega_{N+1} = \left\{ z : x_2 < 0, x_1 \in \mathrm{R}^1 \right\}, \tag{1}$$

under remote loading T in the direction parallel to the boundary of the half-plane.

The coating is modeled as coherently bonded internal strips Ω_r

$$\begin{aligned} &\Omega_r = \left\{ z : H_{r+1} < x_2 < H_r,\ x_1 \in \mathrm{R}^1 \right\}, \\ &H_N = h_N, \quad H_{N+1} = 0, \quad H_r = H_{r+1} + h_r, \quad r = \overline{2,N}, \end{aligned} \tag{2}$$

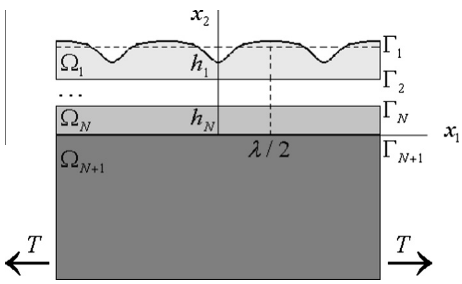

Fig. 1. Multilayer film coating with curved surface.