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Elastic analysis of axially functionally graded rotating thick cylinder with variable thickness under non-uniform arbitrarily pressure loading

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ABSTRACT

A functionally graded rotating thick hollow cylinder with variable thickness and clamped ends is studied semi-analytically under arbitrarily non-uniform pressure on the inner surface. The material properties, except the Poisson's ratio, are assumed to vary with the power law function in the axial direction of the cylinder. By using the first-order shear deformation theory (FSDT) the governing equations are derived. The governing equations are in the form of a set of general differential equations. Given that the FG cylinder with variable thickness is divided into n homogenous disks, n sets of differential equations are obtained. The solution of this set of equations is obtained, applying the boundary conditions and continuity conditions between the layers, radial displacement and stresses. The problem is also solved, using the finite element method (FEM). The obtained results of the disk form multi-layers method (MLM) are compared with those of FEM.

1. Introduction

Functionally graded materials (FGMs) are composite materials which are preferred in many applications such as aerospace and nuclear industries (Keles & Conker, 2011). Unlike layered composites, material properties vary continuously and smoothly throughout a certain dimension in FGMs (Akgoz & Civalek, 2014). A number of papers addressing various aspects of FGM have been published in recent years (Nejad & Fatehi, 2015; Nejad, Rastgoo, & Hadi, 2014; Nejad & Kashkoli, 2014; Simsek & Reddy, 2013; Xue & Pan, 2013).

Given the limitations of the classic theories of thick-walled shells, very little attention has been paid to the analytical and semi-analytical solutions for of these shells. Most of the existing literature deals with the stress or vibration analysis of thin cylindrical shells with variable thickness and is based upon a thin shell or membrane shell theory. However, very little attention has been paid to the analytical solution of thick cylindrical shells with variable thickness, which is due to the limitations of the classic theories of thick-walled shells. Shear deformation theory is a very suitable method for the purpose of calculating stresses and displacements in plates and axisymmetric thick shells (Ghannad, Rahimi, & Nejad, 2013). Assuming the transverse shear effect, Naghdi and Cooper (1956), formulated the theory of shear deformation. The solution of thick cylindrical shells of homogenous and isotropic materials, using the first-order shear deformation theory (FSDT) was derived

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by Mirsky and Hermann (1958). Vekua (1965) constructed a refined theory for shallow shells with variable thickness. Suzuki, Konno, and Takahashi (1981) analyzed the axisymmetric vibrations of a cylindrical shell of which varied in the axial direction by using the thin cylindrical shell theory and an improved thick cylindrical shell theory, using a series solution. Saczuk (1995) presented a more rigorous theoretical approach to describe the elastic behaviour of shell-like systems within the micro-polar continuum. Chen, Coleman, Ma, Morris, and You (2000) used partial fraction expansions and Fourier transforms to derive the elasto-static equations of the shallow circular cylindrical shells. A study was also carried out by Kang and Leissa (2001), in which equations of motion and energy functional were derived for a three-dimensional coordinate system. In another work, Kang (2007) used tensor calculus to derive a complete set of three-dimensional field equations well-suited for determining the behavior of thick shells of revolution having arbitrary curvature and variable thickness. Eipakchi, Rahimi, and Khadem (2003) used the FSDT in order to derive governing equations of thick cylinders with varying thickness and solved the equations using perturbation theory. Duan and Koh (2008) derived an analytical solution for axisymmetric transverse vibration of cylindrical shells with thickness varying monotonically in arbitrary power form due to forces acting in the transverse direction, in terms of generalized hypergeometric function. Nejad, Rahimi, and Ghannad (2009) obtained a complete three-dimensional (3-D) set of field equations developed for elastic analysis of thick shells of revolution with arbitrary curvature and variable thickness along the meridional direction made of FGMs by using tensor analysis. Ozturk and Gulgec (2011) investigated the elastic–plastic deformation of a solid cylinder with fixed ends based on Tresca's yield criterion and its associated flow rule, considering four of the material properties to vary radially according to a parabolic form, made of FGM with uniform internal heat generation.

More recently, Kang (2012) presented a 3-D method of analysis for determining the free vibration frequencies of joined thick conical-cylindrical shells of revolution with variable thickness. Ghannad, Rahimi, and Nejad (2012) investigated the analytical solution for clamped–clamped thick cylindrical shells with variable thickness subjected to constant internal pressure. Arefi and Rahimi (2012) developed a three-dimensional multi-field formulation of a functionally graded (FG) piezoelectric thick shell of revolution with variable thickness, curvature and arbitrary non-homogeneity by using tensor analysis. Using the first-order shear deformation theory and assuming the radially varying elastic modulus, Ghannad and Nejad (2012) presented an analytical solution for displacements and stresses in pressurized thick heterogeneous cylindrical shells. Khoshgoftar, Rahimi, and Arefi (2013) analytically studied elastic analysis of an FG thick-walled cylindrical pressure vessel. Ghannad et al. (2013) obtained an analytical solution for stresses and radial displacement for an FGM clamped–clamped pressurized thick cylindrical shell with variable thickness using shear deformation theory and matched asymptotic method. Nejad, Jabbari, and Ghannad (2014) derived a semi-analytical solution for the purpose of determining displacements and stresses in a rotating cylindrical shell with variable thickness under uniform pressure.

As mentioned above, to the best of the researchers' knowledge, no analytical study has been carried out to date on axially asymmetric FG cylindrical shells with variable thickness. In this study, we analyzed a rotating thick cylindrical shell with variable thickness made of axially FGM subjected to non-uniform internal pressure. The governing equations have been derived, which in the axisymmetric condition and elasto-static state are a system of ordinary differential equations with variable coefficients. These equations normally do not have exact solutions. The multi-layered method (MLM) is used in order to solve the system of equations with variable coefficients. For this purpose, an FG rotating cylindrical shell with variable thickness is divided into n homogenous disks. With regard to the continuity between layers and applying boundary conditions, the governing set of differential equations with constant coefficients is solved. In fact, this technique converts the set of equations with variable coefficient into the set of equations with constant coefficient. Therefore, this method is applicable in different branches of science and engineering. The results obtained for stresses and displacements are compared with the solutions carried out through the finite element method (FEM). Good agreement is found between the results.

2. Problem formulation

In shear deformation theory (SDT), the straight lines perpendicular to the central axis of the cylinder do not necessarily remain unchanged after loading and deformation, suggesting that the deformations are axial and axisymmetric and change along the longitudinal cylinder. In other words, the elements have rotation, and the shear strain is not zero.

Geometry of a thick cylindrical shell with variable thickness h, and the length L, is shown in Fig. 1. The clamped–clamped FG cylinder with variable thickness is subjected to non-uniform internal pressure P. The cylinder is rotating around its axis with constant angular velocity ω.

The location of any typical point m, within the shell element may be determined by R and z, as

$$\begin{cases} m:(r,x), & r=R+z \\ -\frac{h}{2}\leqslant z\leqslant\frac{h}{2} & \&\ 0\leqslant x\leqslant L \end{cases} \tag{1}$$

where z is the distance of typical point from the middle surface. In Eq. (1), R and variable thickness h are

$$\begin{cases} R=r_i+\frac{a}{2}-\frac{x}{2}\tan\beta \\ h=r_i+a-x\tan\beta \end{cases} \tag{2}$$