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# Hybrid materials with negative Poisson's ratio inclusions

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## ABSTRACT

We consider hybrid materials consisting of auxetic (material with negative Poisson's ratio) and non-auxetic phases. The auxetic phase is represented by either spherical or cubic inclusions. We analyse the effective characteristics (the Young's and shear moduli and the Poisson's ratio) computed using either the differential scheme for the effective moduli of composites or the direct finite element simulations. The results are verified through Hashin–Shtrikman bounds. We demonstrate that by creating hybrids from auxetic and non-auxetic phases one can obtain considerable increase in stiffness over the stiffnesses of the phases. The stiffening effect is controlled by the value of the Poisson's ratios of the phases, shape of the auxetic inclusions and their concentration. Depending upon the concentration, the hybrid can be made both auxetic and non-auxetic. Even when the inclusions are cubic the hybrid is still nearly isotropic; it becomes truly orthotropic only when the Poisson's ratio of the auxetic phase is very close to the thermodynamic limit of $-1$. These findings can be applied directly in designing a new class of hybrid materials with enhanced stiffness.

## 1. Introduction

Recent advances in technology and manufacturing have considerably increased requirements for material functionality. On top of common expectations of high strength, stiffness, lightweight and low manufacturing costs, the modern materials are expected to provide a variety of integrated functions, absorb energy, deliver information on structural conditions, resist hostile environment, be durable at various scales and comply with special design restrictions. Often, the conventional materials cannot meet these demands at least to the extent desired. In achieving the goal of creating multifunctional materials with enhanced properties, an alternative approach of combining different materials to achieve the desired properties has gained interest in the last decades. These composites or so-called hybrid materials were defined by Ashby and Bréchet (2003) as 'a combination of two or more materials in a predetermined geometry and scale, optimally serving a specific engineering purpose'. In general, the hybrid materials either exhibit characteristics in between the original constituents or even possess properties, which the constituents do not have. This is achieved by adding geometry and internal architecture as design parameters as well as by extending the traditional range of the properties of the constituents in the negative domain. As a result, a hybrid is created that exhibits properties unachievable by separate constituents.

Extensive literature has been devoted to modelling the mechanical response of hybrid materials. A large number of computational methods have been developed to determine equivalent properties of these composites and to develop a full

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multiscale model to predict their structural response (Kanouté, Boso, Chaboche, & Schrefler, 2009). Essentially, the behaviour of these composites is defined by the material properties and geometry of the constituents along with internal architecture of the hybrid. These hybrids may demonstrate superior structural performance in terms of higher stiffness, strength, toughness, and energy absorption. The hybrids can also show enhanced physical properties such as thermal conductivity and permeability or even unusual behaviour characterised by a negative Poisson's ratio (Gaspar, Smith, & Evans, 2009; Pasternak & Dyskin, 2012; Wei & Edwards, 1998; Wei & Edwards 1999a; Wei & Edwards, 1999b), negative stiffness (Estrin et al., 2004), and negative thermal expansion (Barrera, Bruno, Barron, & Allan, 2005; Ellul & Grima, 2013; Grima, Ellul, Gatt, & Attard, 2012; Miller, Smith, Mackenzie, & Evans, 2009; Takenaka, 2012). The constituents of the hybrid can also be chosen to possess unusual properties enhancing the corresponding effective characteristics. For example, a negative stiffness phase within a composite was shown to produce extreme thermal expansion (Wang & Lakes, 2001), inelastic instability (Jaglinski & Lakes, 2004), extreme damping and the negative Poisson's ratio (Wang & Lakes, 2005). The negative thermal expansion particles were found able to reduce thermal stresses occurring in the hybrid structure (Miller, Smith, Dooling, Burgess, & Evans, 2010). Similarly, the reduction in thermal stresses is achieved in topological interlocking structures (Pasternak, Dyskin, & Shufrin, 2012) whereby the elements that constitute the structure are not bonded together, but rather kept in place by kinematic constraints imposed by the neighbouring elements. Composite laminates with alternating layers possessing either negative Poisson's ratio and positive thermal expansion coefficient or positive Poisson's ratio and negative thermal expansion coefficient enhance the overall negative thermal expansion effect (Lim, 2011). Inclusions with negative shear moduli may in some cases increase and in other cases decrease the effective shear modulus of a composite, whereas the concentration of these inclusions controls the stability of the composite (Dyskin & Pasternak, 2012a; Dyskin & Pasternak, 2012b).

Negative Poisson's ratio materials, also known as the auxetics, make an appealing choice as constituents in the hybrid materials since they naturally enlarge the space of design parameters. It was shown that composite materials made by embedding a negative Poisson's fibre network (Jayanty, Crowe, & Berhan, 2011) or auxetic inclusions of various shapes (Wei & Edwards, 1998; Wei & Edwards, 1999a; Wei & Edwards, 1999b) into a conventional (positive Poisson's ratio) matrix might be auxetics as well. Moreover, it was demonstrated that placing the auxetic inclusions of various shapes in a positive Poisson's ratio elastic isotropic matrix increases the effective Young's modulus even when the Young's moduli of the matrix and inclusions are the same (Pasternak & Dyskin, 2008a, 2008b; Pasternak, Dyskin, & Shufrin, 2010; Wei & Edwards, 1998). The present study is aimed to further examine the properties of hybrids with auxetic inclusions.

This paper is structured as follows. Firstly, we present an overview of the structure and properties of auxetic materials. Secondly, we revisit the hybrid materials consisting of a positive Poisson's ratio matrix with spherical auxetic inclusions and evaluate effective Young's modulus and Poisson's ratio using the analytical differential scheme for composite materials. Thirdly, we present a representative volume element of the hybrid material with randomly distributed cubic negative Poisson's ratio inclusions and analyse it using the finite element method. Then we compare the obtained effective characteristics of the hybrid with the Hashin–Shtrikman variational bounds for two-phase composites (Hashin & Shtrikman, 1963). Lastly, we analyse the mechanical behaviour of the hybrid as a function of the inclusion shapes, volume fraction, and Young's moduli of the constituents.

## 2. Auxetic materials: structure and properties

Auxetic materials are the materials with at least one Poisson's ratio negative. Natural auxetic materials were first found in 1927: negative principal Poisson's ratio of $-1/7$ was observed in a single cubic crystal of Pyrites $FeS_2$ (Love, 1927). Negative Poisson's ratios were reported for cubic monocrystals of many elemental metals (Li, Na, Ka, Rb, Cs, Baughman, Shacklette, Zakhidov, & Stafström, 1998) and non-cubic phases of arsenic, antimony and bismuth (Gunton & Saunders, 1972). Another naturally occurring auxetics is $SiO_2$ in its α-crystobalite phase (Keskar & Chelikowsky, 1992; Yeganeh-Haeri, Weidner, & Parise, 1992). Negative Poison's ratios in some directions were reported for face-centred cubic monocrystals (Baughman, Socrates, et al., 2000; Baughman et al., 1998; Milstein & Huang, 1979). Moreover, Baughman et al. (2000) predicted negative Poisson's ratios for body-centred cubic phases likely existing in white dwarf cores and neutron star outer crusts. Natural isotropic materials with negative Poisson's ratio (a concept usually used in claims of 'unusual' behaviour) are yet to be found. The majority of auxetic materials people deal with are structures whose macroscopic behaviour is described by an elastic (usually anisotropic) continuum with negative Poisson's ratio. Subsequently, the claimed enhanced mechanical properties such as elevated fracture toughness and strength properties are often the properties of the structure unrelated to the sign of the Poisson's ratios (see discussion in Pasternak & Dyskin, 2012). On the other hand, it was theoretically predicted and in some cases experimentally demonstrated that the auxetics exhibit increased shear modulus (compared to the Young's modulus) and enhanced fatigue, indentation and impact resistance (Alderson & Alderson, 2007; Lim, 2013; Liu & Hu, 2010; Prawoto, 2012). It was also shown in Pasternak and Dyskin (2012) that in an isotropic case the negative Poisson's ratio diminishes the influence of body forces associated with the thermal expansion, which in some cases leads to the reduction in thermal stress. The promise of enhanced properties led to an intensive search for new auxetic forms, their computational modelling and practical applications. The latest developments in the area of auxetics have been presented in the special issues of Physica Status Solidi (B) by Gatt, Grima, Narojczyk, and Wojciechowski (2012), Wojciechowski, GRima, Alderson, and Rybicki (2013) and Alderson, Alderson, Grima, and Wojciechowski (2014).