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A robust three-dimensional phenomenological model for polycrystalline SMAs: Analytical closed-form solutions

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ABSTRACT

This paper presents a robust three-dimensional phenomenological model and analytical closed-form solutions to simulate self-accommodation, martensitic transformation and orientation/reorientation of martensite in polycrystalline shape memory alloys (SMAs). The model is developed within the classical framework of thermo-dynamics of irreversible processes and utilizes the volume fractions of self-accommodated and oriented martensite as scalar internal variables and the preferred direction of oriented martensite variants as tensorial internal variable. Linear and exponential interpolation functions are introduced which respectively result in coarse and smooth transitions in stress-induced martensitic transformation. A unified constitutive model is presented for both stress and strain control modes that has the property of completely decoupling the reorientation mechanism from the martensitic transformation mechanism. The time-discrete counterpart of the unified constitutive model is introduced, integrating the evolution equation of martensite reorientation using both implicit backward Euler and explicit forward Euler schemes. Analytical closed-form solutions are derived for the preferred direction of oriented martensite variants and the volume fractions of self-accommodated and oriented martensite. In order to examine capabilities of the developed SMA model as well as the proposed closed-form solutions, two boundary value problems are solved including a thin *NiTi* wire under combined tension–torsion non-proportional loadings and a thin-walled *NiTi* tube subjected to combined internal pressure-tension/compression/torsion-heating paths. In the first problem, the model predictions are compared with the experimental data that shows good correlations. Due to simplicity and accuracy, the model can be used as an efficient and analytic computational tool to analyze structures made of SMAs under multi-axial non-proportional loading histories.

1. Introduction

Shape memory alloys (SMAs) are a new class of multi-functional materials that can undergo diffusionless martensitic phase transformations in response to temperature and/or stress fields and are capable of recovering large inelastic strains. SMAs represent unique features known as pseudo-elasticity (PE) and shape memory effect (SME). These unique characteristics of SMAs together with their good biocompatibility have made them attractive for use in many engineering applications such as sensors, actuators, energy absorption, bio-medical devices and micro-electro-mechanical systems (Lagoudas, 2008).

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The distinctive thermo-mechanical material properties of SMAs such as PE, SME, phase transformations between austenite, oriented martensite and self-accommodated martensite and also orientation/reorientation of martensite, as will be modeled in this paper, are well-known and have been described in many papers and books. For the sake of brevity, these mechanisms are not described here and their details are referred to Lagoudas (2008) and Panico and Brinson (2007).

In order to assess the potential of SMAs in advanced engineering applications especially under multi-axial non-proportional loading conditions, many experimental studies have been conducted over the past two decades. Šittner, Hara, and Tokuda (1995), for the first time, studied the pseudo-elastic tension–torsion behavior of thin-walled tubes made of *CuAlZnMn* under uniaxial tension and biaxial combined tension–torsion proportional/non-proportional loadings. The type of tests consisted of square- and triangle-shaped stress/strain control paths. Helm (2001) and Helm and Haupt (2002) performed biaxial tests on thin-walled tubes under square- and butterfly-shaped strain control loading path. McNaney, Imbeni, Jung, Papadopoulos, and Ritchie (2003) investigated experimentally the combined tension–torsion behavior of thin-walled tubes under different square-shaped tests in the first axial–torsional quadrant varying the aspect ratios of the respective square shape. A large experimental database, made of non-proportional tension-internal pressure tests and biaxial compressive tests on tubular *CuAlBe* specimen, was reported by Bouvet, Calloch, and Lexcellent (2002) to investigate the effect of non-proportionality of the loading path on the pseudo-elastic behavior of SMAs. Several multi-axial proportional/non-proportional experiments on an *NiTi* SMA were performed by Grabe and Bruhns (2009) within a wide temperature range. The results revealed the strong non-linear material response and the response path dependencies, highlighting the presence of reorientation processes for complex loading paths with respect to pseudo-elasticity and the shape memory effect. Recently, a large experimental dataset of combined tension–torsion tests on thin *NiTi* wires has been obtained in frame of SMA Roundrobin modeling initiative carried out within the S3T EUROCORES program (Šittner et al., 2009a).

Some comparisons between simulations with classical models and experimental results under non-proportional loading histories revealed that the predictions of models were not in good correlations with experimental observations. These recent results highlighted that the capability of classical models to handle non-proportional loadings resulting into variants coalescence, variants reorientation, complex combinations of simultaneous forward and reverse transformation and the effect of rhombohedral-phase has to improved (Bouvet, Calloch, & Lexcellent, 2004; Šittner et al., 2009a).

Motivated by accessible experimental results and the fact that SMA devices generally undergo multi-axial non-proportional loadings, a significant research effort has been conducted recently with the aim of developing phenomenological constitutive models that can simulate SMA thermo-mechanical behavior under non-proportional loading conditions. Using the free Helmholtz energy function, Panico and Brinson (2007) proposed a phenomenological three-dimensional (3-D) model to simulate self-accommodation, martensite transformation and orientation/reorientation of martensite variants during multi-axial non-proportional loading histories. A novel feature of the Panico–Brinson model was the treatment of parent phase transformation and martensite variant reorientation as two different physical processes which were governed by distinct evolution laws. In this model, the reorientation was affected by transformation. Panico and Brinson (2008) also modified their original model (Panico & Brinson, 2007) to simulate the response of porous SMAs under cycling loading conditions and the accumulation of irreversible martensite. Motivated by the Panico–Brinson SMA model, Arghavani, Auricchio, Naghdabadi, Reali, and Sohrabpour (2010) formulated a 3-D constitutive model considering no effect on reorientation due to transformation. Their model used a measure of the amount of stress-induced martensite as scalar internal variable and the inelastic strain direction as directional internal variable. Chemisky, Duval, Patoor, and Ben Zineb (2011) developed a phenomenological 3-D SMA model to simulate martensitic transformation, reorientation of martensite and inelastic accommodation of twins in self-accommodated martensite under complex thermo-mechanical loading paths. Moreover, the modeling of minor loops inside the full transformation loop due to partial loading and also the effect of tension–compression asymmetry of the transformation strain magnitude were considered. Motivated by the earlier work of Boyd and Lagoudas (1996), Lagoudas, Hartl, Chemisky, Machado, and Popov (2012) proposed a 3-D SMA model to capture three characteristics of SMA response including the smooth transition in the thermo-mechanical responses, the effect of applied stress level on both transformation strain rate and transformation hysteresis area. A comprehensive phenomenological 3-D model for polycrystalline *NiTi*-based SMAs was formulated by Sedlák, Frost, Benešová, Ben Zineb, and Šittner (2012). The model is capable of realistic simulations of transformation between austenite, rhombohedral-phase and martensite and reorientation of martensite variants, which may simultaneously occur under general thermo-mechanical loading. The effects of tension–compression asymmetry, material anisotropy and martensite stabilization by deformation or evolution of transformation hysteresis with temperature were also considered. Stebner and Brinson (2013) presented an explicit numerical implementation of the SMA model initially proposed by Panico and Brinson (2007). To achieve this implementation, new choices for transformation kinetic law and reorientation limit function were introduced. They proposed two explicit iterative schemes for solving transformation and reorientation equations of their improved model without requiring the implementation of the Jacobian of equations. Recently, Bodaghi, Damanpack, Aghdam, and Shakeri (2013) proposed a phenomenological model to simulate main features of SMA devices under non-proportional loading conditions in which two stress components including one normal and one shear stress are dominant. They solved the coupled governing equations of martensitic transformation and reorientation of martensite using an iterative scheme.

The literature survey reveals that the basic SMA model introduced by Panico and Brinson (2007) has been modified and improved by researchers (Arghavani et al., 2010; Bodaghi et al., 2013; Panico & Brinson, 2008; Stebner & Brinson, 2013) for modeling the thermo-mechanical SMA behaviors under multi-axial non-proportional loadings. To the best of authors' knowledge, all of the existing 3-D SMA models with capability of the reorientation simulation require iterative numerical