International Journal of Engineering Science 82 (2014) 22–27

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

A note on some insights from decoupling the time derivative of an objective tensor

Roney L. Thompson *

LMTA, Department of Mechanical Engineering (PGMEC), Universidade Federal Fluminense, Rua Passo da Patria 156, 24210-240 Niterói, RJ, Brazil

ARTICLE INFO

Article history:
Received 30 November 2013
Accepted 2 May 2014

Keywords:
Scalar-valued tensor function
Objective time derivative tensor decoupling
Natural convected time derivative

ABSTRACT

There are some standard procedures in Mechanics that lead to a final equation that presents an apparent objectivity unbalancing. Generally, this apparent unbalancing has its origin on the material time derivative of an objective quantity. In the present work we analyze three examples: the material time derivative of a scalar-valued function of a second order tensor, the relaxation of a simple viscoelastic material and the Cattaneo equation for the heat flux where this situation occurs. We show that, decoupling the material time derivative of the tensor into a part that is associated to the time derivative of its eigenvalues and another part that is associated to the time derivative of its eigenvectors, only the first part is significant. This conclusion can bring insights for different forms of constitutive assumptions and interpretation of general results.

© 2014 Elsevier Ltd. All rights reserved.

1. Introduction

In the Mechanics literature there are some situations where standard procedures renders to a resulting equation a structure that needs to be further clarified since, apparently, objectivity is not balanced. This does not mean that the original equation is incorrect, but means that an operation which extracts the objectivity structure of the equation, eliminating a non-objective term that is innocuous can be employed. This further step towards writing the equation with objective quantities only can bring some insights to help future work.

1.1. The time derivative of an isotropic scalar-valued function of a symmetric tensor

The time derivative of an isotropic scalar-valued function, a, of a symmetric tensor, $\mathbf{A}$, is frequently used in Mechanics and other fields. As standard examples, we can cite the dependence of the Helmholtz free energy, on the elastic Left Cauchy–Green strain tensor, when the material is undergoing an isothermal process, or the dependence of the Gibbs potential on the symmetric stress tensor, when thermal effects are significant (see Rajagopal & Srinivasa, 2013). If we require the dependence form of $\hat{a}(\mathbf{A})$ to be isotropic, we have that

$$\hat{a}(\mathbf{A}) = \hat{a}\left(\mathbf{QAQ}^T\right), \quad \mathbf{Q} \in \mathcal{O}, \tag{1}$$

* Tel.: +55 21 26295577.
E-mail addresses: rthompson@mec.uff.br, roney@vm.uff.br

http://dx.doi.org/10.1016/j.ijengsci.2014.05.001
0020-7225/© 2014 Elsevier Ltd. All rights reserved.

where $\mathscr{O}$ is the orthogonal group and the superscript T indicates the transpose operation. It can be shown that the isotropic nature of the function makes the form of $\hat{a}(\mathbf{A})$ to be dependent on the invariants of $\mathbf{A}$ solely, i.e.

$$\hat{a}(\mathbf{A}) = \tilde{a}(I_1, I_2, I_3) \equiv \bar{a}(\bar{I}_1, \bar{I}_2, \bar{I}_3), \tag{2}$$

where $I_1 \equiv \mathrm{tr}\mathbf{A},\ I_2 \equiv \frac{1}{2}\left(I_1^2 - \mathrm{tr}\mathbf{A}^2\right),\ I_3 \equiv \det\mathbf{A},\ \bar{I}_i \equiv \mathrm{tr}\mathbf{A}^i$. Hence, the derivative of an isotropic scalar-valued function of a second order tensor with respect to this tensor is given by

$$\frac{d\hat{a}}{d\mathbf{A}} = \left(\frac{\partial\tilde{a}}{\partial I_1} + I_1\frac{\partial\tilde{a}}{\partial I_2}\right)\mathbf{1} - \frac{\partial\tilde{a}}{\partial I_2}\mathbf{A}^T + I_3\frac{\partial\tilde{a}}{\partial I_3}\mathbf{A}^{-T} = \sum_{i=k}^{3} k\frac{\partial\bar{a}}{\partial\bar{I}_k}\left(\mathbf{A}^{k-1}\right)^T, \tag{3}$$

where $\mathbf{1}$ is the identity tensor, the superscript $-T$ indicates the inverse of the transpose of a tensor, and $\mathbf{A}^0 \equiv \mathbf{1}$. Let us consider the case where $\mathbf{A}$ is an objective symmetric tensor. The usual procedure to take the material time derivative of the scalar a is to use the chain rule to obtain

$$\dot{a} = \frac{d\hat{a}}{d\mathbf{A}} \cdot \dot{\mathbf{A}}, \tag{4}$$

i.e. the inner product between $\frac{d\hat{a}}{d\mathbf{A}}$ and the material time derivative of $\mathbf{A}$. The quantity $\frac{d\hat{a}}{d\mathbf{A}}$ is given by one of the two expressions of Eq. (3). This result is pretty standard and can be found frequently in text books (e.g. Truesdell & Noll, 2004). Here we can highlight the point we raise in this short article. When we look to the left side of Eq. (4) we see the material time derivative of an objective scalar, which is objective. However, when we look to the right hand side of Eq. (4) we see the inner product of two tensors: $\frac{d\hat{a}}{d\mathbf{A}}$, which is objective, and $\dot{\mathbf{A}}$ which is non-objective. Hence, the logic behind Eq. (4) can undergo one more step.

Examining Eq. (4) one finds that adding a tensor of the form $\mathbf{A\Omega} - \mathbf{\Omega A}$, where $\mathbf{\Omega}$ is an appropriate skew-symmetric tensor, objectivity balance can be restored. This happens because tensor $\overset{\diamond}{\mathbf{A}} = \dot{\mathbf{A}} + \mathbf{A\Omega} - \mathbf{\Omega A}$ can be objective and the added term is orthogonal to $\frac{d\hat{a}}{d\mathbf{A}}$. However, we must observe that *since* $\mathbf{\Omega}$ *is unrelated to* $\mathbf{A}$ *the resulting tensor* $\overset{\diamond}{\mathbf{A}}$ *is not a* ***part of*** $\dot{\mathbf{A}}$. In other words, one cannot guarantee that $\left\|\overset{\diamond}{\mathbf{A}}\right\| \leqslant \left\|\dot{\mathbf{A}}\right\|$, where $\|\mathbf{T}\|$ is the norm of a generic tensor $\mathbf{T}$. In this sense, the resulting objective balance achieved by the addition of $\mathbf{A\Omega} - \mathbf{\Omega A}$ does not clarifies the point raised. What we need to identify is how we can ***extract*** *from* $\dot{\mathbf{A}}$ *the part that is orthogonal to* $\frac{d\hat{a}}{d\mathbf{A}}$. Therefore, we must guarantee that the resulting objective tensor has a norm which is **lower** than the norm of tensor $\dot{\mathbf{A}}$.

1.2. The relaxation process of a polymeric solution

Let us consider simple constitutive equations for polymeric solutions. Probably, the simplest 3-D constitutive equation that can predict relaxation of a viscoelastic liquid is given by a generalization of the 1-D model proposed by Maxwell. This equation takes the following form

$$\boldsymbol{\sigma} + \lambda_\sigma \overset{\triangledown}{\boldsymbol{\sigma}} = 2\eta\mathbf{D}, \tag{5}$$

where $\boldsymbol{\sigma}$ is the stress, λ_σ is the relaxation time, η is the viscosity, $\mathbf{D}$ is the symmetric part of the velocity gradient and the down triangle indicates an objective time derivative of the tensor. The mostly used Maxwell model is the so called Upper Convected Maxwell model, where the down triangle implies the contravariant convected time derivative operator, defined by

$$\overset{\triangledown}{\boldsymbol{\sigma}} \equiv \dot{\boldsymbol{\sigma}} - \mathbf{L}\boldsymbol{\sigma} - \boldsymbol{\sigma}\mathbf{L}^T, \tag{6}$$

where $\mathbf{L}$ is the velocity gradient. Since the material is viscoelastic, a stress relaxation process takes place when the material experiences a flow cessation, the stress does not vanish immediately. The stress gradually relaxes to a stress-free state. In the flow cessation process (see Bird, Armstrong, & Hassager, 1987), $\mathbf{L}$ and $\mathbf{D}$ become zero in Eqs. (5) and (6). Hence, Eq. (5) becomes

$$\boldsymbol{\sigma} + \lambda_\sigma\dot{\boldsymbol{\sigma}} = \mathbf{0}. \tag{7}$$

The stress relaxation process after the flow is stopped leads to another equation where objectivity is not balanced. The stress, is supposed to be proportional to a non-objective tensor, the stress material time derivative.

1.3. The Cattaneo equation

The Cattaneo equation (see Cattaneo, 1948) is an equation for the heat flux which is based on a correction of the Fourier equation. It is known that the celebrated Fourier equation for the heat flux is an approximation for the case of low frequencies scales and that it leads to a parabolic equation for the temperature evolution. Since it is unreasonable to expect the every disturbance is immediately felt by the whole domain, the evolution of temperature should be hyperbolic. If we consider the particular case where there is no radiation or energy sources and the material considered cannot store energy (no elasticity),

Download English Version:

https://daneshyari.com/en/article/824831

Download Persian Version:

https://daneshyari.com/article/824831

Daneshyari.com