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On the stability of boundary-layer flows over rotating spheroids

A. Samad [a], S.J. Garrett [b,*]

[a] Department of Mathematics, University of Peshawar, Pakistan
[b] Department of Mathematics, University of Leicester, UK

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ABSTRACT

We study linear convective instabilities within of the boundary-layer flows over spheroids rotating in otherwise still fluids. Particular spheroids within the prolate and oblate families are considered, each characterized by an eccentricity parameter, $0 \leqslant e \leqslant 0.7$. Viscous and streamline-curvature effects are included and local analyses conducted at latitudes between 10°–70° from the axis of rotation. Both travelling and stationary convective modes of type I (crossflow) and type II (streamline curvature) are found at each latitude within specific parameter spaces. The results of existing rotating-sphere investigations are reproduced at all latitudes in the limit of zero eccentricity.

In the prolate case, eccentricity is found to have a stabilizing effect on the type I mode at all latitudes and a destabilizing effect on the type II mode at latitudes above 50°. Eccentricity is therefore seen to be a destabilizing influence for low rotation rates (where instability occurs at high latitudes only and the type II mode dominates) and a stabilizing effect for high rotation rates (where instability occurs closer to the pole and the type I mode dominates). This effect is associated with the behavior of the local curvature of the prolate geometry as a function of eccentricity. In the oblate case, eccentricity is found to be universally stabilizing to both mode types at all latitudes. The oblate results demonstrate considerably lower sensitivity to eccentricity than the prolate results.

The most amplified modes are found to travel at 75% of the surface speed at each latitude and eccentricity for both spheroid families. This is consistent with existing theoretical studies of boundary-layer flows over the related geometries of the rotating-disk, -sphere and -cone, and leads to the prediction of *slow* vortices over highly-polished bodies.

1. Introduction

The boundary-layer flow over a rotating disk has served as the foremost model problem for studying transition in fully 3D incompressible boundary layers for over six decades, and has a huge body of associated literature (see (Gregory, Stuart, & Walker, 1955; Hussain, Garrett, & Stephen, 2011; Lingwood, 1995; Malik, 1986; Owen & Rogers, 1989; Reed & Saric, 1989; Saric, Reed, & White, 2003; Smith, 1947; Theodorsen & Regier, 1945; Wimmer, 1988), for example). The rotating-disk flow is of particular interest because it shares many similarities with the flow over swept wings (Reed & Saric, 1989; Saric et al., 2003; Wimmer, 1988) and types of rotating machinery (Hussain, Garrett, et al., 2011; Owen & Rogers, 1989) where laminar-turbulent transition is of significance in engineering design. However, continuing developments in spinning

* Corresponding author. Tel.: +44 0116 2523899.
E-mail addresses: mathematics.peshawar@gmail.com (A. Samad), sjg50@le.ac.uk (S.J. Garrett).

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projectiles, aerofoils and aeroengines has led to a need to understand the onset of laminar-turbulent transition of the boundary-layer flows over rotating cones and spheroids as objects in their own right.

Numerous flow-visualization studies, led by Kohama and Kobayashi, have been published (Kobayashi & Arai, 1990; Kobayashi, Kohama, & Kurosawa, 1983; Kobayashi & Izumi, 1983; Kohama, 1985; Kohama, 2000; Kohama & Kobayashi, 1983; Taniguchi, Kobayashi, & Fukunishi, 1998) which demonstrate an apparent similarity between the properties of rotating-disk, -cone and -sphere flows, as can seen in Fig. 1. In particular, it is clear that as one moves away from the pole within each geometry the flow is first laminar, with co-rotating spiral vortices appearing before their ultimate breakdown and the onset of the turbulent region; increasing the rotation rate of each body acts to move the transitional region closer to the pole. However, geometries other than the disk had received little analytical attention prior to 2002 when Garrett & co-workers commenced a series of studies of the boundary-layer flow over rotating cones and spheres (Barrow & Garrett, 2013; Garrett, 2002; Garrett & Peake, 2002, 2004, 2007; Garrett, Hussain, & Stephen, 2009; Garrett, Hussain, & Stephen, 2010; Garrett, 2010a, 2010b, 2010c; Hussain, Garrett, et al., 2011; Hussain, Stephen, & Garrett, 2011).

This present study should be considered as a generalization of Garrett's previous work on the stability of rotating-sphere flows to the two families (prolate and oblate) of rotating spheroids. Spheroids represent more practically significant rotating nose cones, and this is our motivation. We will see that the laminar flow and stability characteristics of the rotating-spheroid flow are closely related to the rotating-sphere flow. Indeed the sphere is a particular case of either spheroid type with zero cross-sectional eccentricity, and all are related to the rotating-disk flow in the region close to the pole. With this in mind, it is instructive to begin this study with a review of the existing work on the boundary-layer flow over the rotating sphere.

Theoretical studies of the stability of the rotating-sphere flow have been performed by Taniguchi et al. (1998), Garrett and Peake (2002, 2004), Barrow and Garrett (2013), who present local convective and absolute instability analyses of the boundary-layer flow for the sphere rotating both in and out of enforced axial flows and with and without surface mass flux. The studies focused on the calculation of the critical Reynolds numbers and other observable parameters at the onset of instabilities at each latitude. The stability analyses were a natural extension of the previous theoretical work into the steady flow profiles within the boundary layer (Banks, 1965; Banks, 1976; Manohar, 1967) and the stability analyses of the rotating-disk flow due to, for example, Malik (1986), Lingwood (1995). Garrett & Peake and Barrow & Garrett found that local convective-instability analyses could be used successfully to predict experimental observations (Kobayashi & Arai, 1990; Kohama & Kobayashi, 1983; Sawatzki, 1970) of the onset of spiral vortices at each latitude; furthermore, the onset of absolute instability appears to be related to the onset of turbulence (at least for low to moderate latitudes) and this is currently being explored further. As with rotating disks and broad cones, modes of type I (crossflow) and II (streamline curvature) were found to dictate the convective instability at all latitudes over the rotating sphere, and modes of type I and III, the absolute instability. Further information on these instability modes within the related boundary-layer flows can be found in Lingwood (1995), Garrett (2002), Garrett and Peake (2002, 2004, 2007), Garrett et al. (2009, 2010), Garrett (2010a, 2010b, 2010c), Hussain, Garrett, et al. (2011), Hussain, Stephen, et al. (2011) and Garrett (2011).

An interesting experimental observation made by Kobayashi and Arai (1990) was that the co-rotating spiral vortices were fixed on the sphere surface when the rotation rate was large (and transition occurred at low to moderate latitudes), whilst they moved relative to the sphere surface when the rotation rate was smaller (and transition occurred at high latitudes). The relative speed of the slow vortices was always around 76% of the local surface speed of the sphere. This observation was unique to this set of experiments, i.e. slow vortices had not been observed by Sawatzki (1970) or Kohama and Kobayashi (1983) and had not been observed in rotating-disk and cone experiments (see (Kobayashi et al., 1983; Kobayashi & Izumi, 1983; Kohama, 1985), for example). Garrett & Peake originally attempted to clarify the appearance of slow vortices using a method of investigation since denoted *method 1* in Garrett (2010a). This method involves calculating the critical parameters of a set of neutral curves, each pertaining to a different fixed azimuthal wavenumber of disturbance; the azimuthal phase velocity of disturbances, c, is then calculated from the globally-critical parameters and associated with the vortex speed. This approach predicted stationary vortices ($c = 1$) at all latitudes below 66°, where the type I mode dominates. Above this latitude an apparent point of inflection (rather than a global minimum) was found in the distribution of critical Reynolds

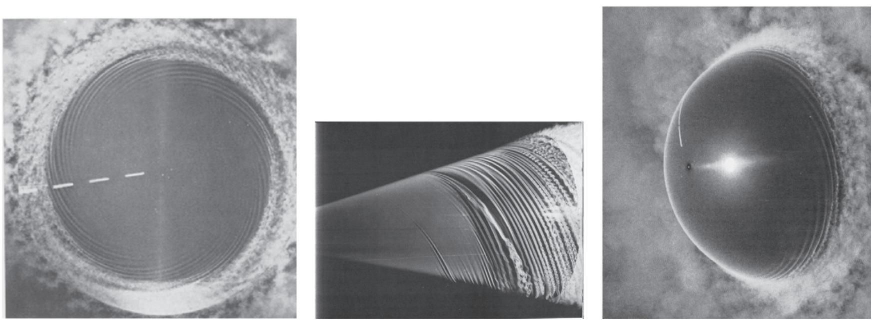

Fig. 1. Boundary-layer flow visualizations over a rotating disk, cone and sphere due to Kohoma, Kobayashi and co-workers Van Dyke (1982).