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Stability of size dependent functionally graded nanoplate based on nonlocal elasticity and higher order plate theories and different boundary conditions

A. Daneshmehr [a,*], A. Rajabpoor [a], M. pourdavood [b]

[a] School of Mechanical Engineering, College of Engineering, University of Tehran, Tehran, Iran
[b] School of Mechanical Engineering, College of Engineering, Iran University of Science and Technology, Tehran, Iran

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ABSTRACT

This article presents a nonlocal higher order plate theory for stability analysis of nanoplates subjected to biaxial in plane loadings. It is assumed that the properties of the FG nanoplate follow a power law form through the thickness. Governing equations and corresponding boundary conditions are derived by using the principle of minimum potential energy. Generalized differential quadrature (GDQ) method is implemented to solve the size dependent buckling analysis according to the higher order shear deformation plate theories where highly coupled equations exist for various boundary conditions of rectangular plates. Some numerical results are presented to study the effects of the material length scale parameter, plate thickness, Poisson's ratio, side to thickness ratio and aspect ratio on size dependent buckling load. It is observed that buckling load predicted by higher order theory significantly deviates from classical ones, especially for thick plates. Also comparing the results obtained from different theories shows that as the material length scale parameter take higher values, the difference between the buckling load resulting from the first order shear deformation plate theory (FSDT), classical theory and higher order plate theory declines.

1. Introduction

Recognition of nanostructure governing equations plays a fundamental role in prediction the effect of various parameters on their functions. But by minimizing the size of the system and becoming comparable to the internal characteristic length scale, the theories based on classical continuum mechanics are not capable of modeling such systems in which size dependent behaviors have been experimentally observed. The atomistic and experimental modeling are computationally exorbitant for modeling the nanostructures with large numbers of atoms. So a conventional form of continuum mechanics that can capture the size effect is required. In the recent decades several nonclassical continuum theories such as strain gradient, couple stress and nonlocal theories that contain additional material length scale have been developed to capture the size effect.

In the first time (Mindlin, 1965) elaborated a higher order strain gradient theory for studying the elastic materials in micro scale. Also Mindlin and Eshel (1968) proposed three equivalent kinds of formulation for the first strain gradient theory. Some special forms of the Mindlin's original formulation are proposed by several researchers (Aifantis, 1992; Gao & Park,

* Corresponding author. Tel.: +98 2188005677; fax: +98 2188013029.
E-mail addresses: Daneshmehr@ut.ac.ir (A. Daneshmehr), Amir_rajabpoor@ut.ac.ir (A. Rajabpoor), Amiremerean@yahoo.com (M. pourdavood).

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2007; Yang, Chong, Lam, & Tong, 2002). Fleck and Hutchinson (1997) developed and reformulated the stated theory and called as the strain gradient theory that contains five higher order material length scale parameters. Kong, Zhou, Nie, and Wang (2009) investigated the static and dynamic responses of Euler–Bernoulli micro-beams using MSGT. They studied the effect of thickness to the size scale parameter ratio of the microbeams on their static deformation and vibrational behavior. Kong et al. (2009) utilized the stated theory to study the static and dynamic behaviors of linear Euler–Bernoulli microbeams. Size dependent static and free vibration analysis of FG Euler–Bernoulli beam model are studied by Kahrobaiyan, Rahaeifard, Tajalli, and Ahmadian (2012) and obtained equations were solved analytically. Sahmani and Ansari (2012) studied the free vibration behavior of FG microplate based on strain gradient and high order plate theory and proposed an analytical solution for simply supported plate.

Mindlin and Tiersten (1962), Toupin (1962), Koiter (1964), and Mindlin (1964)] proposed couple stress theory that contains two higher order material length scale parameters in addition to the two Lame constants. A new version of the stated theory called as modified couple stress theory was proposed by Yang et al. in which size scale parameters were reduced to one size scale parameter (Yang et al., 2002). Employing the modified couple stress theory, Kong et al. (2009) and Park and Gao (2006) studied the static and dynamic behaviors of Euler–Bernoulli microbeams, respectively. Akgöz and Civalek (2011) applied the modified couple stress theory and the modified strain gradient elasticity to the buckling analysis of micro sized beams for various boundary conditions. A size dependent vibration analysis of microplates based on modified couple stress theory is developed by Jomehzadeh, Noori, and Saidi (2011). Thai and Choi (2012) modeled the static bending, buckling, and free vibration behavior of a microplate and proposed an analytical solution for a simply supported plate.

Nonlocal continuum model proposed by Eringen (1972), and Eringen (2002)] are one of the most applied theoretical approaches for the study of nanostructure due to their computational efficiency and the capability to gain more accurate results when the size of the system is in nanoscale (Fotouhi, Firouz-Abadi, & Haddadpour, 2013; Lei, Adhikari, & Friswell, 2013; Rahmani & Pedram, 2014; Reddy, 2010; Thai, 2012). The first study based on nonlocal elasticity theories has been carried out by Peddieson et al. on the analysis of Euler–Bernoulli beam (Peddieson, Buchanan, & McNitt, 2003). They concluded that nonlocal continuum mechanics can potentially play a useful role in nanotechnology applications. Recently, Reddy (2007) applied a version of nonlocal elasticity for formulating a nonlocal version of different beam theories including those of Euler–Bernoulli, Timoshenko, Levinson and Reddy beam theory to analyze bending, buckling and vibration of nanobeams. In his study, different displacement functions are chosen in the first step and then all steps are repeated when deriving beam equations of motion. Also in his study length scale effect cannot be observed (because of considering the constant beam length). Pradhan and Murmu (2009) applied nonlocal elasticity theories to study the stability characteristics of single layer graphene sheets (SLGS) based on classical plate theory (CLPT) and used Levy's approach to solve the governing equations for various boundary conditions of the graphene sheets. Lu, Zhang, Lee, Wang, and Reddy (2007) proposed a nonlocal plate model based on Eringen's theory and derived the basic equations for the classical and the first order plate (FSDT) theories. Aksencer and Aydogdu (2011) derived governing equations of motion of FSDT plates using the nonlocal differential relations of Eringen. Navier type solution was used for simply supported plates and Levy type method was used for plates with two opposite edges simply supported and the remaining ones arbitrary. Babaei and Shahidi (2011) studied the elastic buckling behavior of quadrilateral single layer graphene sheets under biaxial compression employing nonlocal continuum and used the Galerkin method to solve the obtained equations. Narendar (2011) investigated the buckling analysis of isotropic nanoplates using two variable refined plate theory and nonlocal small scale effects and proposed the closed form solution for buckling load of a simply supported rectangular nanoplate. Hashemi and Samaei (2011) proposed an analytical solution for the buckling analysis of rectangular nanoplates and in order to extract characteristic equations of the micro/nanoscale plate under in plane load, the analysis procedure was based on the nonlocal Mindlin plate theory. Due to their novel thermo-mechanical properties, the applications of FGMs have been speared in various industries and engineering applications. Developing of the material technology has led to employ FGMs in micro and nano-sized system and devices such as sensors, nanowires, actuators, atomic force microscopes, thin films to improve their performances (Fu, Du, & Zhang, 2003; Lee et al., 2006; Lu, Wu, & Chen, 2011; Lun, Zhang, Gao, & Jia, 2006; Moser & Gijs, 2007; Rahaeifard, Kahrobaiyan, & Ahmadian, 2009; Stölken & Evans, 1998; Witvrouw & Mehta, 2005). Jung and Han (2013) proposed a model for vibration analysis of sigmoid functionally graded material (S-FGM) nano-scale plate with first-order shear deformation theory. Natarajan et al. investigated size dependent linear free flexural vibration behavior of functionally graded (FG) nanoplates and solve the obtained equations using a finite element approach (Natarajan, Chakraborty, Thangavel, Bordas, & Rabczuk, 2012).

In this paper size dependent buckling analysis of FG nanoplates based on nonlocal elasticity theory is investigated. To gain more accurate results in studying the nanoplate, higher order shear deformation plate theory (HSDT) is required when stocky and short nanoplates are considered. Employing the principle of minimum potential energy the governing equations are obtained. Generalize differential quadrature method (GDQM) is used to solve the governing equations for various boundary conditions. The effect of nonlocal parameter, the size of the FG nanoplate, Poisson's ratio and various boundary conditions, on the dimensionless buckling load and buckling ratio are investigated. These models can degenerate into the classical models if the material length scale parameter and Poisson's ratio are both taken to be zero. Also, the obtained differential equations are analytically solved for a nanoplate with all sides simply supported boundary. Comparison between the results of GDQ and analytical methods reveals the accuracy of GDQ method. At the end some numerical results are presented to study the effects of material length scale parameter, plate thickness, aspect ratio, Poisson's ratio and side to thickness ratio on size dependent buckling load.