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Original paper

Resolution and noise performance of sparse view X-ray CT reconstruction via *Lp*-norm regularization

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A R T I C L E I N F O

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A B S T R A C T

Objectives: Adaptive steepest descent projection onto convex sets (ASD-POCS) algorithms with *Lp*-norm ($0 < p \leq 1$) regularization have shown great promise in sparse-view X-ray CT reconstruction. However, the difference in p value selection can lead to varying algorithm performance of noise and resolution. Therefore, it is imperative to develop a reliable method to evaluate the resolution and noise properties of the ASD-POCS algorithms under different *Lp*-norm priors.
Methods: A comparative performance evaluation of ASD-POCS algorithms under different *Lp*-norm ($0 < p \leq 2$) priors were performed in terms of modulation transfer function (MTF), noise power spectrum (NPS) and noise equivalent quanta (NEQ). Simulation data sets from the EGSnrc/BEAMnrc Monte Carlo system and an actual mouse data set were used for algorithms comparison.
Results: A considerable MTF improvement can be achieved with the decrement of p. *L1* regularization based algorithm obtains the best noise performance, and shows superiority in NEQ evaluation. The advantage of *L1*-norm prior is also confirmed by the reconstructions from the actual mouse data set through contrast to noise ratio (CNR) comparison.
Conclusion: Although the ASD-POCS algorithms using small *Lp*-norm ($p \leq 0.5$) priors yield a higher MTF than do the high *Lp*-norms, the best noise-resolution performance is achieved when p is between 0.8 and 1. The results are expected to be a reference to the choice of p in *Lp*-norm ($0 < p \leq 2$) regularization.

1. Introduction

X-ray cone-beam computed tomography (CBCT) is an important tool for biomedical research and preclinical applications. In spite of the remarkable advantages of CBCT, the exposure risk remains a major concern in clinical practice. To decrease the radiation dose, techniques have been developed to realize the limited-angle CT [1–3] or limited-view CT [4–6]. However, these dose reduction approaches will unavoidably increase the data inconsistency, leading to a challenging task for image reconstruction. To deal with such situation, much effort has been directed to developing iterative image reconstruction algorithms for applications in X-ray CBCT with data inconsistency [7,8].

Iterative reconstruction methods produce good quality images when the projection data is not theoretically sufficient for exact image reconstruction [9,10]. The CBCT reconstruction problem can be solved using iterative algorithms by formulating the data consistency constraint with an additional regularization term [10–13]. One representative algorithm is the adaptive steepest descent projection onto convex sets (ASD-POCS) algorithm [10], which uses the adaptive steepest descent algorithm for total variance minimization—*L1*-norm of the gradient magnitude images, and employs projection onto convex sets algorithm to keep data fidelity. In spite of the advantages of using a *L1* regularization term to solve the sparsity constrained problems, enhancing the sparsity constraint for better imaging performance is still a main concern.

CT image reconstructions from ASD-POCS algorithms using *Lp*-norm ($0 < p < 1$) prior are expected to have higher spatial resolution than those using *L1*-norm or *L2*-norm prior [14–16]. However, quantitative

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frequency comparisons on resolution and noise performance of ASD-POCS algorithms using different *Lp*-norm ($0 < p \le 2$) priors are not sufficient at the present time [10,14,17]. Some papers were based on global evaluation metrics such as signal to noise ratio (SNR), contrast to noise ratio (CNR), and root of mean square error (RMSE) [18–20], and although there have been reports evaluating the spatial resolutions using modulation transfer function (MTF) analysis, they presented results within limited number of *Lp*-norm priors [21,22]. Moreover, relatively few works were conducted on quantitatively comparing both the resolution and noise performance by spatial frequency dependent metrics for the ASD-POCS algorithms with *Lp*-norm ($0 < p \le 2$) regularization. There, the aim of this study is to employ an overall and comprehensive performance comparison of *Lp*-norm ($0 < p \le 2$) priors.

The modulation transfer function (MTF) and the noise power spectrum (NPS) have been widely recognized as the most relevant spatial frequency dependent metrics of resolution and noise evaluation in radiographic imaging. The MTF is a measure of image sharpness and the NPS describes the noise in the image. These two metrics have been introduced into CBCT imaging system evaluation with varying measurement methods [23,24]. MTF and NPS can be further combined to derive the noise equivalent quanta (NEQ), which is a decisive indicator of the signal and noise transfer properties of an imaging system [25].

In CT, methods for measuring the presampling MTF were developed by using an aluminum edge [26] or a tungsten wire [27]. These methods typically use highly dense materials to simulate an infinite impulse to maximize CNR and improve measurement precision. However, these materials are atypical in biomedical research and preclinical applications. Moreover, the use of high-density metallic test object for MTF measurement may introduce metal artifacts that can introduce additional uncertainties owing to their CT artifacts. According to this problem, it is important to use test objects with lower contrast to avoid metal artifacts and yield MTFs relevant to realistic one used in clinical images. A currently widely adopted low contrast phantom for MTF measurement in CT is the American College of Radiology (ACR) computed tomography accreditation program (further denoted as ACR phantom) [28]. The ACR phantom is a solid cylinder phantom constructed primarily from a water-equivalent material. Note that in 2011, low-contrast cylinder test objects similar to ACR phantom were proposed by the American National Standard for MTF measurement of CT [29]. In this standard, the circumference of the cylinder is used as an edge to measure the MTF, and the inner part of it is used to calculate the NPS.

In this work, a Monte Carlo (MC) simulation platform was built employing the EGSnrc/BEAMnrc code system. The CBCT scans were simulated under the condition of different noise levels. Then the simulated and experiment data were employed to quantitatively investigate and compare the resolution and noise performance of *Lp*-norm ($0 < p \le 2$) regularization. Several figures of merit including the MTF, NPS and NEQ, which account for the spatial resolution and the noise performance, were introduced to objectively evaluate the reconstruction performance at different noise levels.

2. Materials and methods

2.1. Virtual CT system

The full MC simulation of a kV CBCT imaging system was carried out with the EGSnrc/BEAMnrc MC code system [30,31]. The component modules in the BEAMnrc simulation consisted of an 'XTUBE' (the X-ray target) and a 'SLABS' (the filter). The SLABS included a filter of 4.0 mm of Aluminum plus 0.1 mm of Beryllium. The tungsten anode target (2 mm thick) was contained in a copper holder, with 9 degree from z-axis (see Fig. 1). Cross sections for all materials in the simulation were generated from the XCOM data set using the PEGS4 code system [32]. The global electron cut-off energy (ECUT) and photon cut-off energy (PCUT) was set to 521 keV and 10 keV, respectively. Rayleigh scattering and bound Compton scattering were switched on, along with atomic relaxations and electron impact ionization. The prediction of X-ray spectrum was derived from the BEAM Data Processor (BEAMDP) program. Specifically speaking, an 80 keV mono-energetic parallel circular electron beam in the simulation was impinged to the target, and then the multiple spectrum containing energy from 0 to 80 KeV was generated. The simulated spectral distribution was presented in the left of Fig. 2. The spectra have been assigned 0.4 keV bin widths and normalized to unit area.

CBCT scanning simulation was performed using egs_cbct, an EGSnrc-based application for CBCT-related calculations. Photons were simulated down to 1 keV and no electron was transported. The latter was achieved by setting ECUT higher than the maximum photon energy used in the MC simulation. CBCT scans of the phantom were conducted with an isotropic point X-ray source modeled by BEAMnrc. The derived spectrum of the X-ray source was presented in Fig. 2. An ideal detector made of 512×512 pixels with a size of 1.5 mm is adopted in the simulation. The X-ray source was located at 630 mm from the phantom's center, and the detector was centered at 970 mm from the source along the line connecting the source and the phantom's center. The source-detector system can be rotated 360° around the phantom.

The phantom used in CBCT scanning simulation was constructed according to the third module of physical ACR phantom, which is a solid cylinder phantom constructed primarily from a uniform water-equivalent material. It is 8 cm in depth and 20 cm in diameter made of solid water, a material in the standard 521ICRU library of EGSnrc (see the right of Fig. 2).

The digital ACR phantom was then reconstructed by the ASD-POCS algorithm using varying *Lp*-norm ($0 < p \le 2$) priors, based on TIGRE, a MATLAB-GPU toolbox for CBCT image reconstruction [33]. Three simulation data sets under three different signal to noise ratios (SNRs) were generated by running different particle histories in EGS simulation for all projection angles. Table 1 shows the simulation and reconstruction parameters. For all *Lp*-norm ($0 < p \le 2$) priors, iteration numbers were set to 300, which were enough to get the convergence.

2.2. Evaluation metrics

Frequency dependent metrics including MTF, NPS and NEQ were calculated for algorithm performance evaluation. MTF was assessed by a radial oversampling methodology [28] from the reconstructed images as illustrated in Fig. 3. The oversampled edge spread function (ESF) was determined on increments of the distance from the edge [28], and then it was fitted with the linear combination of two Gaussian functions and a Boltzmann function to avoid instability [34]. ESF was then differentiated to obtain the line spread function (LSF), from which the MTF was calculated by using the Fourier transform.

To investigate the frequency variation of noise under *Lp*-norm ($0 < p \le 2$) regularization, the NPS was calculated based on ensemble statistics from the ACR phantom images. The NPS utilizes the Fourier transform of noise images to determine the variance of noise power present at each spatial frequency. Noise-only images were obtained by the subtraction of the averaged image from each of 20 central reconstructed slices. Total 720 region of interests (ROIs) in 20 noise-only images were averaged to calculate the 2D NPS distribution. ROIs were created, centered along a circle, as illustrated in Fig. 3 [28]. Finally, the desired NPS profile was generated by averaging 8 radial profiles from center to the edge to avoid fluctuation.

The image quality of the reconstructed ACR phantom was further assessed by the measurement of the noise equivalent quanta (NEQ), which is a measure of the signal to noise ratio of an imaging system. Mathematically, the NEQ can be represented as [29]

$$NEQ(f) = \frac{[S \cdot MTF(f)]^2}{NPS(f)} \tag{1}$$