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On the moving boundary conditions for a hydraulic fracture

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ABSTRACT

This paper re-examines the boundary conditions at the moving front of a hydraulic fracture when the fluid front has coalesced with the crack edge. This practically important particular case is treated as the zero fluid lag limit of the general case when the two fronts are distinct. The limiting process shows what becomes of the two boundary conditions on the fluid front, a pressure condition and a Stefan condition, when the lag vanishes. On the one hand, the pressure condition disappears as the net pressure (the difference between the fluid pressure and the magnitude of the far-field stress normal to the fracture) becomes singular. On the other hand, the Stefan condition, which equates the front velocity to the average fluid velocity, transforms into a zero flux boundary condition at the front. As a consequence, the velocity of the coalesced front does not appear explicitly in the boundary conditions. However, the front velocity can still be extracted from the near-tip aperture field by a nonlinear asymptotic analysis. The paper concludes with a description of an algorithm to propagate the combined front, which explicitly uses the known multiscale asymptotics of the fracture aperture.

1. Introduction

Fluid-driven fractures represent a particular class of tensile fractures that propagate in solid media, typically under pre-existing compressive stresses, as a result of internal pressurization by an injected viscous fluid. Hydraulic fractures are most commonly engineered for the stimulation of hydrocarbon-bearing rock strata to increase production of oil and gas wells (Economides & Nolte, 2000), but there are other industrial applications such as remediation projects in contaminated soils (Murdoch, 2002), waste disposal (Abou-Sayed et al., 1994), preconditioning and cave inducement in mining (Jeffrey & Mills, 2000). Furthermore, hydraulic fractures manifest at the geological scale as kilometer-long vertical dikes bringing magma from deep underground chambers to the earth's surface (Lister & Kerr, 1991; Rubin, 1995), or as subhorizontal fractures known as sills that divert magma from dikes (Pollard & Hozlhausen, 1979).

The design of hydraulic fracturing treatments relies, in part, on our ability to simulate the evolution of the fracture footprint and of the aperture field, as well as of the injection pressure, and to assess the dependence of these quantities on the fracturing fluid rheology, the injection rate, and the rock mechanical properties. However, simulating the propagation of a hydraulic fracture remains a formidable task, even under the ideal assumptions of an isotropic homogeneous linear elastic rock. The challenge stems on the one hand from solving the non-linear, history-dependent, and non-local equations

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governing the flow of a viscous fluid in a deformable permeable channel, and on the other hand from the moving boundary nature of the problem. In principle, there are two moving fronts – the crack edge and the fluid front lagging behind. But under stress conditions that are prevalent in hydrocarbon reservoirs, the lag between the two fronts is virtually non-existent. The two fronts must then be treated as having coalesced in numerical simulators, as a prohibitively dense discretization mesh would be required otherwise to capture the lag.

Paradoxically, it is more challenging to formulate a computational algorithm to propagate the front for the limiting case of zero lag. The complication arises because of the degeneracy of the Reynolds lubrication equation near the fracture tip, where the aperture tends to zero, and also because of a singularity in the leak-off velocity at the tip in permeable rock. As a result, the Stefan condition at the fluid front, which provides a condition on the fluid front velocity when the two fronts are distinct, degenerates into a zero flux boundary condition when the two fronts coalesce. With the front velocity not appearing explicitly in the conditions at the moving front, standard computational algorithms for solving moving boundary problems, such as the volume of fluid method (Voller, 2009) or the level set method (Osher & Sethian, 1988; Sethian, 1999) cannot be used as such.

While considerable effort has been invested in the modeling of hydraulic fracturing since the pioneering work of Khristianovic and Zheltov (1955) (see Adachi, Siebrits, Peirce, & Desroches (2007) and Bunger, Detournay, Garagash, & Peirce (2007) for an extensive list of references, with a particular focus on the Petroleum Industry), the realization that the global solution depends critically on the boundary conditions at the tip and on the details of the near-tip solution has only emerged in recent years. Indeed, when the first models of hydraulic fractures were being developed, the complexity of the problem linked to the existence of a moving boundary and to the degeneracy of the nonlinear equations near the tip was not fully recognized. In these early attempts, analytical solutions for plane strain and radial hydraulic fractures were built based on *ad hoc* assumptions (Abé, Mura, & Keer, 1976, 1979; Advani, Torok, Lee, & Choudhry, 1987; Geertsma & de Klerk, 1969; Nilson, 1986; Nilson & Griffiths, 1983), while numerical models inherited propagation algorithms from dry cracks based on linear elastic fracture mechanics (Advani, Lee, & Lee, 1990; Clifton, 1989; Clifton & Abou-Sayed, 1979; Shah, Carter, & Ingraffea, 1997; Sousa, Carter, & Ingraffea, 1993; Vandamme & Curran, 1989); these algorithms unwittingly forced a behavior in the tip region that was not always appropriate for the spatial resolution of the mesh (Lecampion et al., 2013). Furthermore, the non-linearity of the equations implies that it is possible to find multiple volume-conserving and equilibrated fracture width and fluid pressure fields associated with different fracture footprints at a given time. The role of the boundary conditions is to select the appropriate fracture width, fluid pressure, and fracture footprint combination.

Recent research efforts have led to a series of accurate benchmark solutions for simple hydraulic fracture geometries (with zero lag): plane strain (Adachi, 2001; Adachi & Detournay, 2002, 2008; Garagash, 2006a, 2006b; Garagash & Detournay, 2005; Hu & Garagash, 2010) and radial (Bunger, Detournay, & Garagash, 2005; Madyarova & Detournay, 2013; Savitski & Detournay, 2002). These solutions, which have also been partially verified by laboratory experiments (Bunger, 2005; Bunger et al., 2007), provide rigorous tests for numerical algorithms (Lecampion et al., 2013), and also are forcing a re-examination of the tip boundary conditions and of the importance of the solution in the vicinity of the fracture front.

Motivated by the recent resurgence of papers on the modeling of hydraulic fractures (Carrier & Granet, 2012; Chen, 2012; Damjanac, Detournay, Cundall, & Varun, 2013; Gordeliy & Detournay, 2011; Gordeliy & Peirce, 2013a, 2013b; Linkov, 2012; Mishuris, Wrobel, & Linkov, 2012; Mohammadnejad & Khoei, 2013; Hunsweck, Shen, & Lew, 2013; Zhou & Hou, 2013; Zhang & Jeffrey, 2012), we carefully re-examine here the conditions at the fluid and fracture fronts. Furthermore, we use tip asymptotic analysis to highlight the change in the required boundary conditions in the singular limit in which the fluid and the fracture fronts coalesce. The crack front velocity can then only be extracted from a non-linear asymptotic analysis of the solution in the tip, a challenging task in itself because of the multiscale nature of the tip solution. We conclude by describing an algorithm that exploits the tip asymptotics to both locate the free boundary and to determine the front velocity, and which is capable of achieving an accurate solution on a relatively coarse mesh.

2. Mathematical formulation

2.1. Problem definition and assumptions

We consider the propagation of a planar fracture, driven by the injection of a fluid in a rock medium, see Fig. 1. The hydraulic fracture is, in principle, characterized by the two distinct moving fronts that evolve with time t: one is the crack edge $\mathcal{C}_c(t)$ and the other is the fluid front $\mathcal{C}_f(t)$, which is contained inside $\mathcal{C}_c(t)$. The contour $\mathcal{C}_c(t)$ defines the crack footprint $\mathcal{A}_c(t)$, while $\mathcal{C}_f(t)$ defines the fluid-filled fracture domain $\mathcal{A}_f(t) \subseteq \mathcal{A}_c(t)$. Under large far-field stress conditions, the lag between the crack edge and the fluid front becomes negligible (Garagash & Detournay, 2000), and the two fronts effectively coalesce to a single front denoted by $\mathcal{C}(t)$, which encompasses the crack domain $\mathcal{A}(t)$.

The fluid is viewed as being injected from a point source because the characteristic dimension of $\mathcal{C}_f(t)$ and thus of $\mathcal{C}_c(t)$ is much larger than the dimension of the source. The injection point serves as the origin for the vector $\boldsymbol{x}$ defining the position of any point in the fracture plane.

The main focus of this paper is on the nature of the boundary conditions at the moving front $\mathcal{C}(t)$ that results when the lag vanishes. Before addressing this question, we first formulate the complete set of equations for the case when $\mathcal{C}_c(t)$ is distinct from $\mathcal{C}_f(t)$. A complete formulation of the problem requires that the governing equations, the boundary conditions on $\mathcal{C}_c(t)$