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Qualitatively new models of microinhomogeneous media obtained by homogenization

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ABSTRACT

Averaged effective models for inhomogeneous media with constituents of very different properties, in particular, composites, mixtures, and porous materials are considered in this paper. It is known that effective properties of composites and mixtures may differ from their components properties not only quantitatively but also qualitatively. The paper aims to demonstrate examples of appearance of qualitatively new models as a result of homogenization procedure. Media consisting of qualitatively similar components are considered. The following effective models are described: models with higher derivatives that describe dispersion of waves in composites consisting of elastic components while there is no dispersion of waves in individual components; models of compressive media for incompressible elastic materials with pores; models with long-range memory or with additional internal parameters for mixtures of viscous fluids as well as for composites consisting of elastoplastic components; models for sound propagation in mixtures of fluids that are different depending on relations between the inhomogeneity length scale and parameters linked with fluids viscosity and heat conductivity.

The paper is a brief review of the results obtained by N.S. Bakhvalov and the author.

1. Introduction

This paper is devoted to description of the behavior of inhomogeneous media with constituents of very different properties, in particular, composites, mixtures, and porous materials. Let d be a characteristic length scale of the inhomogeneity and L be the global length scale, i.e., the dimension of the body or the typical wavelength in a dynamic problem. If $\varepsilon = \frac{d}{L} \ll 1$, we speak about microinhomogeneous media. In this case it is worth to introduce an effective homogeneous medium whose behavior is close to that of the original inhomogeneous medium in some sense. If the local physical properties and the geometrical structure are known, the properties of the effective medium are determined by certain averaging.

One of methods for the derivation of averaged equations has been developed in the mathematical theory of differential equations with rapidly oscillating coefficients. The problem is stated as follows. Let the equations describing the behavior of the inhomogeneous medium be known. These equations contain a microscale parameter ε. We should construct equations that do not contain any microscale parameter and whose solution is close to the solution of the original equations in a certain norm. For media with periodic structure, the method of two-scale asymptotic expansions to derive averaged equations and approximately determine the local fields had been developed (see, e.g., Bakhvalov, 1974; Bakhvalov & Panasenko, 1984;

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Bensoussan, Lions, & Papanicolaou, 1978; Sanchez-Palencia, 1980). The procedure based on this method is called homogenization and models arising in homogenization are called effective models.

It is known that effective properties of composites and mixtures may differ from properties of their components not only quantitatively but also qualitatively. This is observed even when all components are qualitatively similar. One of examples is dispersion of waves in elastic composites while there is no dispersion in separate components. In this paper (which in fact is a review) we demonstrate appearance of qualitatively new models as a result of homogenization procedure. Media consisting of qualitatively similar components are considered. We don't give a full review of the papers containing results of such kind, speaking mainly about results obtained by N.S. Bakhvalov and by the author. We describe the main features of the derived effective models. Strict estimations of closeness of solutions of the effective equations to solutions of the original ones can be found in the referenced papers.

2. Homogenization algorithm

The homogenization algorithm consists of the following steps. In addition to slow variables x_i with the characteristic variation scale L we introduce the fast variables $y_i = x_i/d$ with the characteristic variation scale d. Below, we use dimensionless variables and assume that $L = 1$, and $d = \varepsilon \ll 1$.

A solution u to the problem for original equations is considered as a function of independent variables $x_i,\ y_i,\ t$. Then, we expand u in the asymptotic series

$$u = v + \varepsilon v_1 + \cdots$$

with the coefficients that are periodic in y_i. This series is substituted into the original system of equations and boundary conditions. Equating the coefficients at equal powers of ε, we obtain the so-called problems on unit cell. Solution to these problems and the averaging over the periodicity cell yield averaged equations and an algorithm for the approximate calculation of the local fields. An important part of the procedure is the strict estimation of the closeness of the solutions of averaged equations to those of original equations.

3. Mixtures and composites consisting of elastic components

The application of the homogenization algorithm to mixtures and composites consisting of linearly elastic components with the elasticity moduli a_{ijkl} (which are different for different components) shows that, in the zero-order approximation with respect to ε, the effective medium also is a linearly elastic medium with the elasticity moduli $\bar{a}_{ijkl}$ and the average density $\bar{\rho}$. The effective moduli are calculated in terms of the solutions to the problems on unit cell. The latter problems are the problems of static extension and shear of the periodicity cell along the coordinate axes in the absence of body forces when certain periodicity conditions are specified on the sides of the cells.

Taking into account higher order terms in ε, we obtain qualitatively new models for the effective media – media with stresses depending on the higher order derivatives of displacements with respect to time and coordinates (Bakhvalov & Panasenko, 1984). In particular, presence of higher order derivatives explains the dispersion of waves in composites consisting of linearly elastic components. The coefficients at the higher derivatives can be calculated by solutions of the unit cell problems. We investigated the main terms responsible for wave's dispersion in locally anisotropic microinhomogeneous media, as well as in locally anisotropic plates and bars analytically and numerically. Dependence of wave's velocity on their length for structures with various types of symmetry had been studied, see the review in Eglit (2010).

Certain general properties of the effective equations of higher order for inhomogeneous nonlinear elastic media with periodic structure had been investigated. One of the questions was: is it possible that a composite made of dissipation-free components would exhibit dissipation in a certain order of approximation in ε? The following statements show that for a medium with periodic structure the answer is "no" (Bakhvalov & Eglit, 1990; see also Berdichevsky, 1983, 2009).

The equations for periodic elastic medium are the Euler equations for the functional

$$I = \int_t \int_x (T - U)dxdt, \quad U = U(y_i, x_i, \nabla_i \mathbf{u}),$$

$$T = \frac{1}{2}\rho(y,x)u_t^2, \quad x = \{x_1, x_2, x_3\}, \quad y_i = \frac{x_i}{\varepsilon}, \quad \nabla_i \mathbf{u} = \frac{\partial \mathbf{u}}{\partial x_i} + \frac{1}{\varepsilon}\frac{\partial \mathbf{u}}{\partial y_i}.$$

In Bakhvalov and Eglit (1990), an averaging procedure was constructed that leads to the equations that are also the Euler equations for a certain functional

$$\hat{I}(\mathbf{v}) = \int_t \int_x (\hat{T} - \hat{U})dxdt,$$

$$\hat{T} = \left\langle \frac{1}{2}\rho(x,y)(\mathbf{u}(\mathbf{v}))_t^2 \right\rangle, \quad \hat{U} = \langle U(x, y, \mathrm{grad}\ \mathbf{u}(\mathbf{v}))\rangle, \quad \mathbf{u}(\mathbf{v}) \sim \mathbf{v}(x) + \sum_{n=1}^{\infty} \mathbf{v}_n(x,y)\varepsilon^n,$$