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Original paper

Optimization-based region-of-interest reconstruction for X-ray computed tomography based on total variation and data derivative

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ABSTRACT

Region-of-interest (ROI) and interior reconstructions for computed tomography (CT) have drawn much attention and can be of practical value for potential applications in reducing radiation dose and hardware cost. The conventional wisdom is that the exact reconstruction of an interior ROI is very difficult to be obtained by only using data associated with lines through the ROI. In this study, we propose and investigate optimization-based methods for ROI and interior reconstructions based on total variation (TV) and data derivative. Objective functions are built by the image TV term plus the data finite difference term. Different data terms in the forms of L1-norm, L2-norm, and Kullback–Leibler divergence are incorporated and investigated in the optimizations. Efficient algorithms are developed using the proximal alternating direction method of multipliers (ADMM) for each program. All sub-problems of ADMM are solved by using closed-form solutions with high efficiency. The customized optimizations and algorithms based on the TV and derivative-based data terms can serve as a powerful tool for interior reconstructions. Simulations and real-data experiments indicate that the proposed methods can be of practical value for CT imaging applications.

1. Introduction

Computed tomography (CT) [1] offers a powerful tool to detect the inner structures of objects. Since its advent in the 1970s, CT has played a significant role in imaging science and technology. CT imaging has been widely used in medical imaging and industrial inspections, and related theories and techniques have achieved great progress[2]. Utility-driven applications have motivated the evolution of core CT techniques. Among the various techniques in CT imaging, image reconstruction algorithms play one of the most important roles. In practical use, one may often face the demand to reconstruct images of a central part inside the whole image support from truncated projections. These problems are often referred as interior or local tomography. Interior and local tomography is a specific form of region-of-interest (ROI) reconstruction. In order to obtain high reconstruction accuracy, many of the reconstruction algorithms, such as the Feldkamp-Davis-Kress (FDK) [3] algorithm and the simultaneous algebraic reconstruction technique[4], are usually performed with full image support, where the detector should be wide enough. However, local tomography reconstruction for very large objects, especially when performed on minimum image support, which is decided by the field of view (FOV), often faces data truncation. Incomplete and truncated projections make reconstructions difficult to tackle. Thus, many works have focused on these problems, and some of them have made important progress.

One conventional wisdom is that the exact reconstruction of the interior problem cannot be obtained by only using data associated with lines through the ROI [5]. Reliable reconstructions require a wide detector and adequate projection views. Nevertheless, high-accuracy reconstructions using truncated projection data are highly desirable. The fundamental relationship [6] between the Hilbert transform of an image along a line and the differentiated projection data of the image has been revealed and stimulated the interest of researchers. Over the past years, analytical methods for tackling reconstructions with truncated projections have been proposed [7–9]. Some ideas considering adding priori knowledge of a tiny sub-region [10–12] have proven that a unique solution can be driven through the differentiated back-projection (DBP) method. For interior tomography, the iterative projection onto convex sets[13] is incorporated in the DBP framework, but this step inevitably increases the computational burden. Singular value decomposition-based methods[14,15] avoid iterative processing and provide images with potential utilities. The use of analytic continuation can uniquely and stably solve the interior problem if a sub-region inside an ROI is known. However, such a known sub-region is not always readily available, and it is even impossible to find in some cases.

Despite the fact that reconstruction with truncated projection data is a severely ill-posed problem, the advent of compressive sensing (CS)

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has brought new ideas for processing this problem. Wang et al. [16–18] have proven that "if an object under reconstruction is essentially piecewise constant, a local ROI can be exactly and stably reconstructed via the total variation minimization." Thus, methods based on the piecewise constant assumption of images have driven some total variation (TV)-based methods [19,20] and applications [21,22]. Since the work of Wang et al., the theory of CS-based interior reconstruction has served as a potentially useful tool for developing practical algorithms. However, many of the CS-based methods mentioned in the present study take full image support during reconstruction. Reconstruction with only a minimum image support exactly covering the ROI cannot avoid facing the peculiarity caused by data truncation. Inspired by the lambda tomography (LT) [5] for local tomographic imaging, Pan et al. have proposed finite derivative-based reconstruction methods [23–25]. The comparison in Ref. [23] shows that the derivatives of the acquired and estimated projections are highly similar. This property can be utilized to form data fidelity in the reconstruction programs. In the work of Pan's group, the Chambolle–Pock primal–dual (CPPD) algorithm [26–28] has been taken [24] in the design of a working algorithm to investigate and evaluate the finite derivative-based reconstruction, which shows impressive results. However, it may face difficulties for sparse-view projections because it is not particularly designed for this situation.

Despite progresses in ROI reconstruction (or interior reconstruction), improved optimization programs and efficient algorithms are still needed. In this study, we proposed and evaluated the variants of a derivative-based data fidelity optimization program for ROI reconstruction. The optimization is solved by the alternating direction method of multipliers (ADMM) [29,30], which has already been used in several interesting applications [31–33]. In the present work, stable, efficient, and easy coded algorithms are derived in the framework of ADMM. Results show that specially customized optimizations based on TV combining derivative-based data terms, along with carefully designed algorithms by ADMM, can serve as an efficient tool for interior and ROI reconstructions for full and reduced views.

The outline of this paper is as follows. In Section 2, basic data model, optimization programs, and corresponding algorithms are introduced. In Section 3, simulations using the conventional Shepp–Logan (SL) phantom and an inverse-crime study are considered to test the properties of the proposed model and algorithms. In Section 4, experiments based on real CT data are performed to verify the capabilities of the proposed methods. Necessary discussions of the proposed models and algorithms are presented in Section 5. Finally, in Section 6, a brief summary of the work is presented with comments on its possible extension.

2. Optimization methods for CT IIR using derivative-based data fidelity

We introduce the optimization designed to solve the problem of reconstruction from truncated data in ROI reconstruction. We first describe the data acquisition settings and then define the basis of the proposed method. The specific optimizations and the corresponding algorithms are then presented.

2.1. Data acquisition model

In this study, we consider the typical fan-beam CT setting with circular scanning trajectories as the imaging platform. A typical CT scanner mainly consists of an X-ray source, a detector, and the corresponding mechanical gantry system and data processing system. The simplified geometry of the scanning system is described in Fig. 1, which characterizes some essential geometry terms, e.g., distance of source to the rotation axis (SOD), distance of source to the detector (SDD), source orbit (red solid circle), and FOV (depicted by greed area). Modeling the system serves as the basis of image reconstruction. In the discrete-to-discrete (DD) view, CT data acquisition can be written as

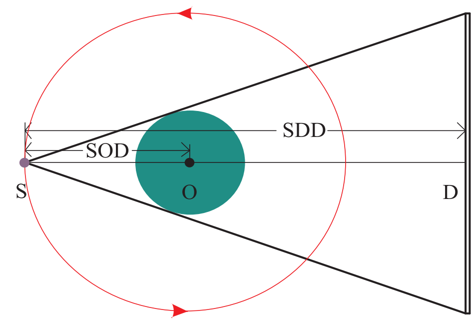

Fig. 1. Simple sketch of CT scanning geometry.

$$\mathbf{Wf} = \mathbf{g}, \tag{1}$$

where $\mathbf{f} \in \mathbf{R}^N$ is the image vector composed of voxel coefficients, $\mathbf{W} \in \mathbf{R}^{M\times N}$ is the system matrix generated by voxel projection, and $\mathbf{g} \in \mathbf{R}^M$ is the data vector containing the measured projection data. The projection matrix $\mathbf{W}$ is an approximation of the data acquisition of the imaging system, and the approximation can be of relatively high degrees when the discretization is fine enough.

In this study, we consider the situation where the projection data are truncated along the horizontal direction of the detector panel. A 2D demonstration of truncated acquisition is shown in Fig. 2. Under this situation, the reconstruction problem is interesting from different viewpoints. If we focus only on the reconstruction inside the bold and solid circle (shown in the left most part of Fig. 2), then the projection data may be adequate but contaminated by the line integral through an object outside this region. From another point of view, if we consider the reconstruction on the full support of the image, i.e., the region marked by the dashed circle in Fig. 2, then the projection data are incomplete. Regardless of the specific situation, both problems are not easy to tackle. When the reconstruction is conducted with under-sampled views, the problem becomes even more troublesome.

2.2. Local operators for discrete data fidelity

For reconstruction with minimum image support and truncated projections, a method that suppresses data truncation must be developed. Theoretically, for the DD imaging model, obtaining an exact local operator that can remove the effects of data truncation is impossible. Nevertheless, some theoretical conclusions driven from the continuous situation can shed some light on the design for the DD imaging model. A reconstruction procedure is called local if the computation of function f at point x requires only values of line (under 2D situation) or plane (under 3D situation) integrals, which meet an arbitrarily small neighborhood of x. However, for 2D parallel scanning, the Radon inversion formula is not a local computation:

$$f(x) = \frac{1}{4\pi}\int_{S^1} (\mathrm{H}g')(\theta,x\cdot\theta)\mathrm{d}\theta, \quad g = \mathrm{R}f, \tag{2}$$

where S^1 is the back-projection path, g' is the first-order derivative of projection data, H is the Hilbert transform, and R is the Radon transform. LT is a conventional method to obtain a local inversion formula in image domain $\mathbf{R}^2$. In LT inversion, the Hilbert transform is dropped, and the order of the derivative of projection is raised by 1, which converts (2) into the form of

$$(\Lambda f)(x) = \frac{-1}{4\pi}\int_{S^1} g''(\theta,x\cdot\theta)\mathrm{d}\theta. \tag{3}$$

In view of the Fourier domain, LT means applying a cone filter on the frequency data, i.e., $(\Lambda f)^\wedge(\xi) = |\xi|\hat{f}(\xi)$. Theoretically and obviously, images generated by LT are not equivalent to CT images because of the presence of cone filter in the frequency domain. Nevertheless, LT keeps