International Journal of Engineering Science 80 (2014) 74–89

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Compression of a hyperelastic layer-substrate structure: Transitions between buckling and surface modes

Yang Liu, Hui-Hui Dai *

Department of Mathematics, City University of Hong Kong, 83 Tat Chee Avenue, Hong Kong, China

ARTICLE INFO

Article history:
Received 6 January 2014
Accepted 13 February 2014
Available online 18 March 2014

Keywords:
Layer-substrate
Buckling mode
Surface mode
Transition
Asymptotic analysis

ABSTRACT

This paper studies the bifurcation behaviors of a hyperelastic layer bonded to another hyperelastic substrate of finite thickness subjected to compression. We aim at revealing some interesting transitions between different bifurcation modes as the geometrical parameters vary. A linear bifurcation analysis is carried out for obtaining the bifurcation condition in the framework of exact theory of nonlinear elasticity. This condition, in the form of a determinant with complicated elements, contains a few parameters, and here the task is to analyze it to determine different behaviors. From the critical stretch curves, it is found that there are two mode types for the layer: buckling mode and wrinkling mode. By further considering the eigenfunction, three types of modes for the substrate are identified, including buckling mode, buckling-surface mode and wrinkling-surface mode. A careful analysis is carried out to determine the parameter constraints for each type of modes. In particular, three critical thickness ratios and two critical aspect ratios of the layer are found. As a result, we manage to classify the plane of the aspect ratio of the layer and the thickness ratio into six domains for different mode types and whose boundaries determine where the transitions of mode types take place. Finally, an asymptotic analysis with double expansions for each unknown is carried out to give the explicit formulas for the critical mode number and the critical stretch (which also give an improvement on the existing results for a layer coated to a half-space). Also, simplified relations for those critical thickness ratios and aspect ratios are derived. The asymptotic results also reveal some interesting insights, e.g., why the Poisson's ratio has little effect and in a wrinkling mode the critical stretch is almost independent of the layer thickness.

© 2014 Elsevier Ltd. All rights reserved.

1. Introduction

Layered structures are widely used in real world and have attracted a lot of interest in both elasticity and engineering. A layer coated to a substrate can be used to model the structures in a large number of engineering and biological situations, such as submarine coatings, surface processing, growth of fruits and vegetables, etc. Recent efforts in this subject were motivated by the understanding of wrinkles generation in human skin and fruits and by applications to the template and assembly of materials. Accordingly, many experiments were carried out for catching the behaviors of layered structures from a variety of prospectives. For two-layer structures, most attention was focused on a stiff layer coated to a soft substrate. For example, due to the thermal expansion mismatch or moisture changes (see Cai, Breid, Crosby, Suo, & Hutchinson,

* Corresponding author. Tel.: +852 27888660; fax: +852 27888561.
E-mail addresses: yliu82-c@my.cityu.edu.hk (Y. Liu), mahhdai@cityu.edu.hk (H.-H. Dai).

http://dx.doi.org/10.1016/j.ijengsci.2014.02.020
0020-7225/© 2014 Elsevier Ltd. All rights reserved.

2011) or the mechanical properties mismatch (see Pocivavsek et al., 2008), for a thick enough substrate, various wrinkles like square checkerboard mode, hexagonal mode, triangular mode, herringbone mode, can occur in such structures. If one only considers the lateral side of the structure, wrinkles are observed, i.e., the geometry of the layer is sinusoidal with a large mode number. The observation from experiments showed that, for a wrinkling mode, the deformation was mainly located in the layer and interface. Moreover, for sandwich panels, under compression at two ends, a global buckled shape can occur if the core (center layer) is soft and the two surface layers are thin (see Wadee, 1998).

Theoretically, the stability of layered structures has been considered by many authors (see Bigoni & Ortiz, 1997; Biot, 1965; Cai & Fu, 1999; Cai & Fu, 2000; Dorris & Nemat-Nasser, 1980; Lee, Triantafyllidis, Barber, & Thouless, 2008; Ogden & Sotiropoluos, 1996; Shield, Kim, & Shield, 1994) and the references cited therein. For a thin layer coated to a substrate, many authors adopted the Von Karman plate theory (which only takes into account the geometrical nonlinearity) to approximate the layer. By assuming a proper form of the displacement field and minimizing the total energy of the layer-substrate, the equations for the amplitude of wrinkles can be obtained (see Cai et al., 2011; Chen & Hutchinson, 2004; Huang & Suo, 2002; Huang, Hong, & Suo, 2005). Also, different substrates have been considered both experimentally and theoretically. For example, in Audoly (2011) the author considered a fluid substrate and in Pocivavsek et al. (2008) the corresponding experiment was done by putting a thin film on water. For those substrates, under compression wrinkles can also occur in the film/layer. From the perspective of exact theory of nonlinear elasticity (with both geometrical and material nonlinearity), the linear stability properties have been considered by many authors when the substrate is a half-space (see Cai & Fu, 1999; Cai & Fu, 2000; Lee et al., 2008; Ogden & Sotiropoluos, 1996; Shield et al., 1994). In Bigoni and Ortiz (1997), the authors also considered the situation that there is a debonding between the layer and substrate and found that it can promote instability. In Cai and Fu (1999), the authors obtained the amplitude equation for a wave propagation problem by asymptotic methods. In Cai and Fu (2000), the authors did an asymptotic stability analysis in the thin film limit and successfully got the leading order solution for two cases of the stiffness ratio. In Lee et al. (2008), the effect of material properties varying with depth was also studied. Although a half-space substrate can be used to approximate a thick enough substrate, in a real situation the substrate is always of finite thickness. As far as the authors are aware of, the influence of such a finiteness on the bifurcation behavior has not been examined, at least in the framework of nonlinear elasticity. Intuitively, one would think that if the substrate is thin enough the whole structure can also be in a global buckled mode. Then, naturally one wonders when the transition between wrinkling and buckling modes occurs. Such an issue together with other related ones will be examined in the present work by using a model of a hyperelastic layer bonded to a hyperelastic substrate of finite thickness (see Fig. 1). It should be mentioned that transitions between buckling and barreling modes have been found in a nonlinearly elastic layer/plate (see Beatty & Dadras, 1976; Beatty & Hook, 1968; Dai & Wang, 2010) and a nonlinearly elastic cylindrical tube (see Goriely, Vandiver, & Destrade, 2008).

For the present problem, one can establish the bifurcation condition by a linear bifurcation analysis through an incremental theory of nonlinear elasticity without much difficulty. Nevertheless, the finite thickness of the substrate causes an extra geometrical parameter appearing in this condition, and analyzing it providing a complete picture how the bifurcation behaviors change as the geometrical parameters vary is less trivial. Actually, for the mode types of the substrate it is necessary to further consider the eigenfunction, and it turns out that there are three mode types for the substrate: buckling mode, buckling-surface mode and wrinkling-surface mode. For the latter two types the displacement decays exponentially along the thickness direction, a character of a surface instability. We also mention that bending a rubber block or an elastic layered structure can also lead to a surface instability (see, respectively Gent & Cho, 1999; Roccabianca, Gei, & Bigoni, 2010). Considerable efforts are then made to establish the geometrical constraints for each of the three mode types. Finally, three critical thickness ratios of substrate and layer and two critical aspect ratios of the layer are obtained. As a result, we can characterize the geometrical parameter plane into six domains, whose boundaries represent the transitions between modes. Then we have a complete picture about the bifurcation behaviors.

To deduce more explicit formulas for the critical stretch and the critical mode number, we consider the thin film limit. As a novelty, we introduce double asymptotic expansions for the each of the two unknowns to deduce two-term solutions in a consistent manner, which also improve the asymptotic results in literature for a film coated to a half-space. In particular, in a wrinkling-surface mode case, it is found that the critical stretch is independent of the aspect ratio of the layer and the

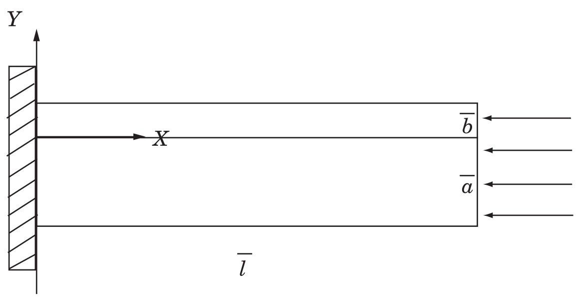

Fig. 1. The geometry of the problem.

Download English Version:

https://daneshyari.com/en/article/824879

Download Persian Version:

https://daneshyari.com/article/824879

Daneshyari.com