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Nonlinear effects in a plane problem of the pure bending of an elastic rectangular panel

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Dedicated to Prof. Leonid M. Zubov on the occasion of his 70th birthday.

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ABSTRACT

The paper presents a modification of the semi-inverse representation of the pure bending deformation of a prismatic panel with rectangular cross-section that is suitable for the method of successive approximations (or Signorinis expansion method). This modification was used to investigate with an accuracy to second order terms the plane problem of pure bending for three different models of nonlinearly elastic behavior: harmonic material, Blatz and Ko material, Murnaghan material. It was found that the typical diagram of bending has maximum point followed by falling region. To study the stability of the bent panel the bifurcation approach was used. Some results about the position of bifurcation points at the loading diagram depending on the material and geometrical parameters are presented.

1. Introduction

The bending as well as tension and torsion is one of the main type of a deformation of construction elements so many problems on different types of bending were thoroughly investigated. Within the framework of the linear elasticity the problem of bending of a prismatic body was solved by Saint-Venant (1856). Later the problem was generalized to different body shapes, loading types and material properties in the case of small strains.

As to the finite strains the title problem has been studied extensively for incompressible materials for many years, dating back at least to the contributions of Rivlin (1949). For incompressible materials the transformation describing bending of a rectangular bar into the cylindrical panel is universal, see e.g. Saccomandi (2001), that means it satisfies the equilibrium equations for every type of isotropic constitutive relation. Different aspects of the stability of the incompressible bent panel was studied by Triantafyllidis (1980); Haughton (1999), Coman and Destrade (2008) and Destrade, Gilchrist, and Murphy (2010).

The corresponding compressible problem has received less attention, although there is a clear analysis in the book by Ogden (1984), which includes an analytical solution for a particular class of materials. Exact nonlinear solution of a plane problem on pure bending of a bar in the case of large strains based upon the harmonic type material model was published in the classical treatise of Lurie (1970). Analytical expression describing bar bending for the Blatz and Ko compressible material model was presented by Carroll and Horgan (1990). There are also nice discussions in the papers (Aron & Wang, 1995; Bruhns, Xiao, & Meyers, 2002).

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The spatial nonlinear theory of pure bending of a prismatic bar was developed by Zelenina and Zubov (2000). In their works three-dimensional problem of bending was reduced to the two-dimensional nonlinear boundary value problem for the plane region having the shape of the bar cross-section. Linearized problem on the bending of a pre-stressed prismatic bar within the framework of the superimposing the small strains on the large one was solved by Zubov (1985).

As to an investigation of the effects of the second order during bending the authors usually analyze the case of initially bent bodies (see e.g. Batra, Dell'Isola, & Ruta, 2005). Such "ignoring" the case of a straight rod may be connected with some specific feature of traditional semi-inverse relations describing the bending deformation. These relations are valid for every non-zero value of bending parameter (e.g. the panel curvature) but don't admit direct transition to the undeformed state of the panel. This makes the application of the successive approximations method difficult because of the absence of the first term in the expansion.

Present paper first of all clearly defines the problem. On the basis of analysis of the exact solution of the bending problem for harmonic material a modification of the semi-inverse representation of the bending deformation is proposed. The feasibility of this modification is confirmed by investigation second order effects for three different models of nonlinearly elastic behavior: harmonic material, Blatz and Ko material, Murnaghan material. Analytical expression for the relative change of the panel thickness is obtained. It can be useful for experimental determination of the elastic modulus of the second order.

The numerical analysis of this boundary value problem for a function that describes the radius of a point in the deformed state shows that the diagram of the bending (the graph of the bending moment with respect to angle of bending) has a maximum point followed by a falling segment. So the question about the stability of the obtained solutions arises. To study the stability the bifurcation approach based upon the linearization of the equilibrium equations was used. The bifurcation point is assumed to be identical with the point of existence of the non-trivial solutions of linearized uniform boundary value problem. The influence of geometrical parameters of the panel upon the bifurcation points distribution along the loading diagram was studied numerically.

2. Rectangular block bending. General relations

Let us consider nonlinearly elastic panel with rectangular cross-section $-a/2 \leqslant x \leqslant a/2, -h/2 \leqslant y \leqslant h/2$ in the undeformed configuration (Fig. 1 (a)). We denote Cartesian coordinates in the reference configuration as x, y, z and choose cylindrical coordinates r, φ, z as coordinates in the actual (deformed) state. Correspondent unit vectors $\mathbf{e}_x$, $\mathbf{e}_y, \mathbf{e}_z$ and $\mathbf{e}_r, \mathbf{e}_\varphi, \mathbf{e}_z$ are also shown in Fig. 1. We assume no displacement in the z-direction though the stresses in that direction are not zero that corresponds to the case of the plane strain (Lurie, 1970). The bending of the panel is described by following semi-inverse representation (see Fig. 1)

$$r = r(x), \quad \varphi = By, \tag{1}$$

where $B = \gamma/h = \text{const}$. Transformation (1) turns the rectangular cross-section into the sector of the circular ring so it describes the bending of the panel. At this bending the cross-sections $y = \text{const}$ stays plane suffering the distortion along the direction $\mathbf{e}_r$ that is characterized by $r(x)$ and rotation around vector $\mathbf{e}_Z$ through an angle of By. The value $\rho = 1/B$ is a distance between the origin and the position of the rectangle's center of gravity in the deformed state.

The equilibrium equations in the volume in the absence of body forces have the form

$$\text{Div}\,\mathbf{P} = 0. \tag{2}$$

Here Div is the divergence operator at the reference configuration, $\mathbf{P}$ – non-symmetric Piola–Kirchhoff stress tensor, constitutive relation for which can be written as follows

$$\mathbf{P} = \frac{\partial W(\mathbf{F})}{\partial \mathbf{F}}. \tag{3}$$

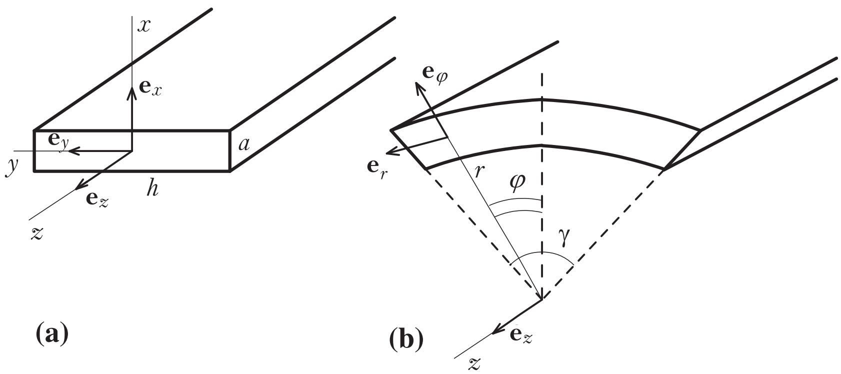

Fig. 1. Plane strain of the rectangular block at the pure bending: (a) reference configuration; (b) deformed state.