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Effective thermal conductivity of a composite with thermo-sensitive constituents and related problems

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ABSTRACT

The paper focuses on the problems related to homogenization procedure for thermal conductivity of a composite with thermo-sensitive constituents (i.e. materials with conductivities dependent on temperature). It is shown that in the simplest case of non-linearity, when thermo-sensitive inhomogeneity is embedded into linearly-conductive matrix, Eshelby theorem (Eshelby, 1957, 1961) does not hold – remotely applied uniform heat flux yields a non-uniform one inside the inhomogeneity. However, in the case when both – the matrix and the inhomogeneity – are thermo-sensitive and their conductivities are proportional to each other, Eshelby theorem for heat flux does hold. For materials of this type, the concept of resistivity contribution tensors is formulated that allows one to generalize the main homogenization schemes used in micromechanics for this special case of non-linearity. The requirement of proportionality holds for many important material systems, including, in particular, porous thermo-sensitive materials or ones reinforced with superconductive inhomogeneities.

1. Introduction and general concepts

In the present paper, we focus on the problems related to calculation of effective thermal conductivity of a heterogeneous material with thermo-sensitive constituents (i.e. constituents with temperature-dependent properties). It is motivated by practical needs, especially in various aerospace and microelectronics applications, where materials are subjected to very high variation of temperature. It is well known (Carslaw & Jaeger, 1959), that thermal conductivity of most materials strongly depends on temperature (Fig. 1 illustrates this dependence for several materials). Due to that micromechanical modeling of thermal conductivity of heterogeneous materials under assumption of constant properties of the constituents may be an over-simplification yielding loss of relevance for many applications.

Many results on non-linear conductivity of particulate composites are obtained in the context of *electrical properties* with governing equations connecting current density $\boldsymbol{j}$ and electric field $\boldsymbol{E}$ as

$$\boldsymbol{j}(\boldsymbol{x}) = \boldsymbol{\sigma}(\boldsymbol{E}) \cdot \boldsymbol{E}(\boldsymbol{x}) \tag{1.1}$$

or for composite dielectrics with constituents showing non-linear dependence between electric displacement and electric field. In the series of works of Hui, Cheung, and Kwong (1997), Hui and Wan (1996), Hui, Woo, and Wan (1995), Wan,

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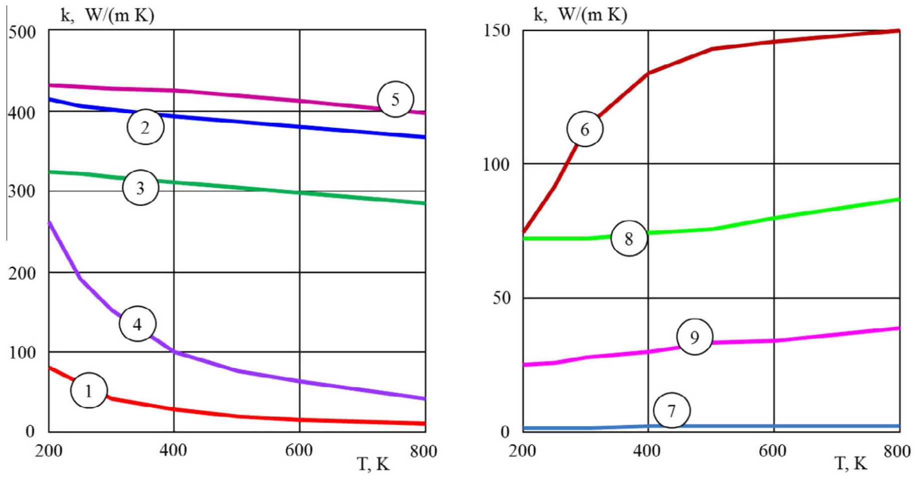

Fig. 1. Thermal conductivity of some materials (in W/m · K) as function of temperature (in *K*): 1 – alumina; 2 – copper; 3 – gold; 4 – silicon; 5 – silver; 6 – brass; 7 – carbon; 8 –palladium; 9 – uranium (data from Hewitt, 2008).

Lee, Hui, and Yu (1996), Yu and Gu (1993) the authors developed self-consistent effective media and effective field theories for non-linear composites that is analogous to those in the linear theory. Blumendeld and Bergman (1991) developed a perturbation theory to calculate effective properties of composite dielectrics. Their approach has been further advanced by Yu, Wang, Hui, and Gu (1993). Ponte-Castaneda, Btton, and Li (1992) proposed a methodology for estimating the effective properties of nonlinear dielectrics based on a variational principle and derived Hashin–Shtrikman type bounds for two-phase non-linear isotropic dielectric composites. Snarskii and Buda (1997) and Snarskii and Zhemirovskiy (2002) used method of local linearization, similar to iterative method in non-linear mechanics, to find effective electrical conductivity of a particulate composite. The main feature of (1.1) that makes it possible to generalize methods of homogenization developed for linear materials to this case of non-linearity is dependence of $\boldsymbol{\sigma}(\boldsymbol{E})$ on the electric field, not on potential of the electric field. As it will be shown later, this type of dependence allows certain generalizations of Eshelby's theory to the non-linear inhomogeneity embedded into a linearly-conductive matrix.

Situation is completely different in the case of composites with *non-linear thermal conductivity* when temperature gradient ∇T and heat flux $\boldsymbol{q}$ are connected by expressions

$$\nabla T(\boldsymbol{x}) = \boldsymbol{R}[T(\boldsymbol{x})] \cdot \boldsymbol{q}(\boldsymbol{x}) \tag{1.2}$$

where proportionality coefficient depends on temperature rather than temperature gradient.

Some attempts to account for temperature dependence of the thermal conductivities of composites have been done for periodic systems. Telega, Tokarzewsky, and Galka (2001) derived bounds for effective conductivity of non-linear two-phase material of periodic structure. Tokarzewski and Andrianov (2001) proposed a methodology to apply linear analysis to non-linear problems in the theory of periodic composites with temperature dependent conductive properties. Galka, Telega, and Tokarzewsky (2001) derived Hashin–Shtrikman type bounds and Golden–Papanicolaou integral representation for quasi-linear composites. They also obtained explicit formulas for the case of layered structures. However, the general theory of conductivity of the thermo-sensitive heterogeneous material cannot be developed without solving the single inhomogeneity problem. In the present paper we focus on a special class of binary materials, for which a single inhomogeneity problem can be solved. Namely, composites with isotropic constituents for which thermal conductivities are proportional to each other. The ultimate goal of the present paper is to generalize typical micromechanical approximate schemes of homogenization for a particulate composite of this type.

2. Single inhomogeneity problem

2.1. Formulation of the problem

We consider an infinite isotropic body (matrix) with thermal conductivity $k_0(T)$ dependent on temperature T. It contains a single inhomogeneity (filler particle) occupying domain Ω (Fig. 2); isotropic conductivity of the inhomogeneity $k_1(T)$ also depends on the temperature. The homogeneous boundary conditions (Hill, 1963) are assumed: the remotely applied heat flux $\boldsymbol{q}^0$ is uniform everywhere in the absence of the inhomogeneity. We also assume perfect contact between the matrix and the inhomogeneity:

$$T(x)|_{\partial\Omega_+} = T(x)|_{\partial\Omega_-}; \quad k_0(T)\frac{\partial T(x)}{\partial n_j}\bigg|_{\partial\Omega_+} = k_1(T)\frac{\partial T(x)}{\partial n_j}\bigg|_{\partial\Omega_-} \tag{2.1}$$