International Journal of Engineering Science 80 (2014) 173–188

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

A variational model based on isogeometric interpolation for the analysis of cracked bodies

Massimo Cuomo *, Loredana Contrafatto, Leopoldo Greco

Department of Civil and Environmental Engineering, University of Catania, Italy

A R T I C L E I N F O

Article history:
Received 2 January 2014
Accepted 13 February 2014
Available online 5 April 2014

Keywords:
Fracture
Isogeometric analysis
Strong discontinuity
Cohesive mechanics
Crack tracking

A B S T R A C T

A variational model for the analysis of crack evolution is presented. The method considers strong discontinuities that evolve according to the principles of cohesive fracture mechanics. A novel isogeometric interpolation scheme is presented that, differently from previous proposals, inserts the fracture modifying the blending properties of the interpolation. A method for tracking the discontinuity is also proposed, based on a local distortion of the parametrization of the geometry obtained determining the position of the control points of the isogeometric interpolation as solution of a suitable minimization problem.

1. Introduction

In this work we turn our attention to a variational formulation for a deformable body with a discontinuity embedded in it, that can evolve from an initial configuration, that may or may not contain a gap. The aim of the study is to explore the ability of the formulation to implement a computational procedure for the analysis of fracture evolution in a continuous body. A cohesive model of fracture is adopted, introducing an energy dissipation function of the discontinuity, that can be viewed as a regularization of the Griffith functional. As a matter of fact cohesive fracture can be effectively modeled using strong (or weak) discontinuous kinematics, that in the FE context is introduced breaking the standard C^0 continuity either introducing zero thickness elements (Ciancio, Carol, & Cuomo, 2007), or using node to node separations, as in Discontinuos Galerkin approaches (Noels & Radovitzky, 2008). Element free formulations have been used for higher order damage theory that can be viewed as a regularization of discontinuous fracture mechanics (Yang & Misra, 2010). In all these cases the simplicity of the model has as payoff a considerable increase in the computational complexity.

A successful approach to the FE modeling of discontinuities is the so called Strong Discontinuity Approach, that introduces enhanced interpolation functions in the elements containing a displacement jump (Belytschko, Moës, Usui, & Parimi, 2001; Melenk & Babuška, 1996; Moës, Dolbow, & Belytschko, 1999; Oliver & Huespe, 2003; Oliver, 1996; Oliver, Cervera, & Manzoli, 1999; Ventura, Xu, & Belytschko, 2002). If the additional degrees of freedom are local the geometric connectivity of the domain does not change. The method, in spite of its simplicity, has still many non solved critical points, particularly related to the definition of the kinematics, to the tracking of the discontinuity, to the presence of stress locking. Usually the direction of the discontinuity path is defined by some criterion based on the stress state at the Gauss points, and the discontinuity crosses entirely the element (Alfaiate, Pires, & Martins, 1997; Alfaiate, Simone, & Sluys, 2003; Dias-da Costa, Alfaiate, Sluys, & Júlio, 2009; Linder & Armero, 2007). Stress locking can arise as a consequence, so that fine meshes are required. Moreover, mesh independency is still a matter of discussion in many cases (Oliver, Huespe, & Dias, 2012).

---

* Corresponding author. Tel.: +39 0957382263; fax: +39 0957382298.
E-mail addresses: mcuomo@dica.unict.it (M. Cuomo), lcontra@dica.unict.it (L. Contrafatto), leopoldo.greco@virgilio.it (L. Greco).

http://dx.doi.org/10.1016/j.ijengsci.2014.02.017
0020-7225/© 2014 Published by Elsevier Ltd.

Roughly summarizing, computational analysis of fracture evolution in computational mechanics requires (a) a kinematics that admits discontinuity in the displacement field, (b) some activation criterion, (c) an algorithm for detecting the fracture path. Sufficiently regular discontinuous kinematic approximations are used in order to be compatible with numerical schemes, although more complex enhancements have been proposed that include singularities (Zi & Belytschko, 2003). Questions relating the fulfillment of boundary conditions and the compatibility of the interpolations for the regular and the singular parts are still open.

Several activation criteria have been proposed in the literature, either for static or for dynamic crack growth (as an example of the latter, see for instance Andreaus & Baragatti (2009) where some experimental comparisons are also provided). Stress based criteria are the simplest, and they use a limit surface defined in the space of the surface tractions, similarly to Leon's criterion, thus requiring the determination of the plane over which the activation function is maximum (Dias-da Costa, Alfaiate, Sluys, & Júlio, 2010). The evolution of localized damage that has been proved to occur prior to fracture is completely neglected in this way, so more sophisticated criteria have been proposed that employ a damaging continuum and introduce the discontinuity corresponding to the loss of ellipticity of the energy functional (Oliver & Huespe, 2004). In order to avoid the numerical difficulties connected with the sudden change of the structure of the boundary value problem, intermediate criteria have been proposed (Moës, Stolz, Bernard, & Chevaugeon, 2011; Patzák & Jirásek, 2004). Generally, the connection of those methods with classical Griffith energetic criterion was not completely stated, mainly because Griffith criterion cannot predict the onset of a crack in an initially uncracked body.

Griffith's criterion states that a crack grows if the energy release rate equals the fracture toughness of the material (Griffith, 1920). The statement is a local stability criterion coupled with an activation criterion, in the sense that the crack is stable and can grow if the variation of the free energy of the elastic body is zero for an increase of the crack along a prescribed path:

$$\frac{dH}{dS} = \frac{d}{dS}(\Phi(S) + G_c(S)) = 0 \tag{1}$$

where $\Phi(S)$ is the stored energy of the equilibrium configuration with a given crack area S, and G_c is the fracture energy, that, according to Griffith's idea, is given by $G_f dS, G_f$ being the toughness of the material. It has been pointed out (Francfort & Larsen, 2003) that the criterion (1) is a local criterion, involving only infinitesimal extensions of the crack.

In order to extend Griffth's criterion to the determination of the crack path, Francfort and Marigo, in a celebrated paper, proposed a global stability criterion, stating that at each instant the crack path is obtained minimizing the internal energy together with the deformation field (Francfort & Marigo, 1998). They proved that the formulation implies Griffith's criterion, although the variational condition is stronger (Negri & Ortner, 2008). Later the formulation was extended to include cohesive fracture, external loads (Bourdin, Francfort, & Marigo, 2008; Dal Maso, Francfort, & Toader, 2005) and a computational procedure based on a suitable regularization of the discontinuous field was presented (Bourdin, Francfort, & Marigo, 2000). A similar approach was proposed by Miehe, Welschinger, and Hofacker (2010), who in addition modified the energy functional originally introduced in Francfort and Marigo (1998) assuming the evolution of the crack surface to be fully dissipative in nature. In these works the crack path is obtained as the limit of a series of problems, in the sense indicated by Ambrosio and Tortorelli (1990). Non local and phase field approximations have been proposed (Hakim & Karma, 2009; Miehe et al., 2010; Miehe, Hofacker, & Welschinger, 2010).

In the absence of a variational formulation tracking the crack path requires the introductions of rules variously motivated (Ortiz & Pandolfi, 1999). In the context of strong discontinuities, the mesh itself in some way influences the choice of the path. One of the causes of the problem is the low continuity level present in FE interpolations, that introduces jumps on the stress field as well as on the discontinuity path.

The Strong Discontinuity Approach based on the Element with Embedded Discontinuities (EED) has proved in a vast literature to be a very efficient methodology for the analysis of quasi brittle fracture. Thus a computational implementation of the variational theory of fracture starting from the tools of EED appears very appealing, although several points need to be investigated. The paper aims to provide some contributions to this project, that range from a revisited formulation of the variational problem to its numerical implementation.

Specifically, first a general multifield variational principle for a body with discontinuities will be provided, as an extension of the Hu–Washizu three-fields principle, with the addition of an enhanced discontinuous displacement field and of a properly defined dissipation function on the discontinuity surface, and it will be proved that it is equivalent, under some restrictions, to the variational principle of Francfort and Marigo. The functional depends both on the mechanical fields and on the position of the discontinuity, that is modeled by means of a collocation operator that yields the geometry of the body. Both fields are conveniently interpolated, so that an optimization procedure can be carried out on finite dimensional spaces. It is interesting to point out that already in 1988, in advance to the work of Francfort and Marigo, Stumpf, Weichert and Le proposed a variational model of fracture whose variation with respect to the deformation and to the position fields allows to obtain the displacements and the position of the discontinuity (Le, Stumpf, & Weichert, 1989; Stumpf & Le, 1990). Their work faced many aspects of the problem within the framework of Griffith's theory. Although the present proposal bears many similarities, we develop our theory within the framework of cohesive mechanics, especially focusing on its numerical implementation in a novel numerical scheme.

An essential point in the paper is the implementation of an algorithmic scheme based on a discontinuous isogeometric interpolation that, contrarily to standard FEM, guarantees a higher continuity level, both for the displacement and for the geometry, of class C^{p-1}, p being the degree of the interpolation. Isogeometric analysis is becoming widely used for problems