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Dielectric elastomers: Asymptotically-correct three-dimensional displacement field

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A R T I C L E I N F O

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A B S T R A C T

Asymptotically-accurate dimensional reduction from three to two dimensions and recovery of 3-D displacement field of non-prestretched dielectric hyperelastic membranes are carried out using the Variational Asymptotic Method (VAM) with moderate strains and very small ratio of the membrane thickness to its shortest wavelength of the deformation along the plate reference surface chosen as the small parameters for asymptotic expansion. Present work incorporates large deformations (displacements and rotations), material nonlinearity (hyperelasticity), and electrical effects. It begins with 3-D nonlinear electro-elastic energy and mathematically splits the analysis into a one-dimensional (1-D) through-the-thickness analysis and a 2-D nonlinear plate analysis. Major contribution of this paper is a comprehensive nonlinear through-the-thickness analysis which provides a 2-D energy asymptotically equivalent of the 3-D energy, a 2-D constitutive relation between the 2-D generalized strain and stress tensors for the plate analysis and a set of recovery relations to express the 3-D displacement field. Analytical expressions are derived for warping functions and stiffness coefficients. This is the first attempt to integrate an analytical work on asymptotically-accurate nonlinear electro-elastic constitutive relation for compressible dielectric hyperelastic model with a generalized finite element analysis of plates to provide 3-D displacement fields using VAM. A unified software package 'VAMNLM' (Variational Asymptotic Method applied to Non-Linear Material models) was developed to carry out 1-D non-linear analysis (analytical), 2-D non-linear finite element analysis and 3-D recovery analysis. The applicability of the current theory is demonstrated through an actuation test case, for which distribution of 3-D displacements are provided.

1. Introduction

Electro-elastomers are a subset of electroactive polymers that exhibit large mechanical strain due to the electric field. Electro-elastomers are referred to as dielectric elastomer actuators when used in actuation mode. Dielectric elastomers consist of a dielectric hyperelastic material sandwiched between two deformable electrodes. These have potential applications in prosthetic devices, endoscopic surgery, compliant robots, adaptive structures, pumps, valves, inflation of membranes and sensors (Bar-Cohen, 2003; Carpi & de Rossi, 2004; Goulbourne, Mockensturm, & Frecker, 2005, 2007; Pelrine, Kornbluh, & Joseph, 1998). A schematic representation of the operation of a dielectric hyperelastic structure is shown in Fig. 1. Applying

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a voltage through complaint electrodes, the dielectric material becomes polarized with microscopic dipoles distributed throughout the body. In this way, the material responds to the external electric field with a, so-called, induced electric field opposing the external field. Energy is stored in this induced field and interacts with the mechanical energy such that the total energy is minimized when the system is in equilibrium. The 'stress' developed in the material by the induced field is called the Maxwell stress. Electro-mechanical coupling occurs due to energy contribution from the product of Maxwell stress and Green strain. Simple elastic models for uniform uniaxial deformations of dielectric elastomer actuators have been formulated by Pelrine et al. (1998) and Kofod (2001). The mathematical modeling of the properties of such materials, however, is at an early stage of development, partly because of a shortage of sufficient experimental data that can be used for materials characterization (Dorfmann & Ogden, 2006). Rogovoy (2001) shows that material slight compressibility can be effectively taken into account in the case of high hydrostatic pressure or highly confined material. In all other situations the application of the incompressible and nearly incompressible material theories gives practically the same results.

To simplify the analysis and provide certain insights, mechanical effects from mechanical loads were studied independently first (Burela & Harursampath, 2012), then coupling of electrical and mechanical effects under electric field were studied. In the current work, an asymptotically-accurate nonlinear analysis of electro-elastomer membrane structures is developed without any *ad hoc* assumptions. The problem is both geometrically and materially nonlinear. The geometric nonlinearity is handled by allowing for finite deformations and generalized warping while the material nonlinearity is incorporated through dielectric hyperelastic material model. Electrical effect is derived through Maxwell–Faraday electrostatics. The development is based on the Variational Asymptotic Method (VAM), first proposed by Berdichevsky (1979). VAM is a synergy of variational and asymptotic methods without their individual handicaps. Asymptotics are applied to functionals rather than to differential equations, thus making systems of governing equations and boundary conditions consistent and less prone to errors. Taking advantage of moderate strains and very small thickness-to-smallest wavelength ratio as the required small parameters, the development begins with three-dimensional nonlinear electro-elasticity and mathematically splits the analysis into a one-dimensional (1-D) through-the-thickness analysis and a two-dimensional (2-D) plate analysis. The through-the-thickness analysis provides nonlinear constitutive relation between the generalized 2-D strain and stress tensors for the plate analysis and a set of recovery relations to express the three-dimensional (3-D) displacement field in terms of 2-D variables determined from solving the equations of the plate analysis. Asymptotically-correct analytical expressions are derived for warping functions and stiffness coefficients (2-D constitutive law). Consistent with the 2-D nonlinear constitutive law, a 2-D nonlinear plate theory is developed and a corresponding nonlinear finite element analysis program was developed. Since there is no literature available to compare the results of non-prestreched membranes with nonlinear material characterization, current theory is demonstrated for a test case in the form of distribution of three-dimensional fields.

2. Kinematics

A systematic dimensional reduction of dielectric hyperelastic membranes is carried out using the Variational Asymptotic Method (VAM). These membranes can be modeled as 2-D structures because of their relatively large in-plane dimensions compared to their thicknesses. More specifically, the order of magnitude of the thickness to the in-plane dimensions is $h/l \ll 1$. Before applying VAM, kinematics of membranes is formulated in line with the procedure adapted for isotropic hyperelastic plates (Burela & Harursampath, 2012). Undeformed and deformed configurations of an arbitrary material point $Q(x_1, x_2, x_3)$ of the 3-D continuum is illustrated in Fig. 2. The reference surface is described by two coordinates, x_α (Here and throughout the formulation, Greek indices assume values 1 and 2 while Latin indices assume 1, 2, and 3. Dummy indices are summed over their range, except where explicitly indicated) and x_3 is the normal coordinate. Position vectors of $Q(x_1, x_2, x_3)$ in the undeformed and deformed configurations are denoted by $\hat{\mathbf{r}}$ and $\hat{\mathbf{R}}$, respectively, from any point O and given by

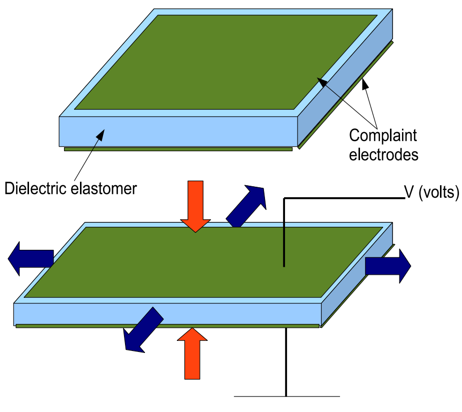

Fig. 1. Schematic representation of operation of dielectric hyperelastic structure.