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Modified couple stress theory applied to dynamic analysis of composite laminated beams by considering different beam theories

M. Mohammad-Abadi, A.R. Daneshmehr *

School of Mechanical Engineering, College of Engineering, University of Tehran, Tehran, Iran

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ABSTRACT

In this study, by using the modified couple stress theory, the vibration analysis of composite laminated beams in order of micron is developed. It should be mentioned that this theory is capable to capture the size effect by considering the material length scale parameters unlike the classical continuum theories. The Hamilton's principle is applied to obtain the governing equations and boundary conditions of micro composite laminated beams. By considering three beam models, i.e. Euler–Bernoulli, Timoshenko and Reddy beam models, the differences between them and the effect of shear deformation are studied. This is the first study that introduces the couple stress-curvature relation for Reddy beam model properly. Furthermore, three boundary conditions, i.e. hinged–hinged, clamped–hinged and clamped–clamped and four types of lamination, i.e. [0,0,0], [0,90,0], [90,0,90] and [90,90,90] are investigated. Using generalized differential quadrature (GDQ) method, the governing equations are numerically solved and natural frequencies are obtained. Also, the governing equations are analytically solved for hinged-hinged boundary condition by employing the Fourier series expansions. Comparison between results obtained by GDQ method and analytical solution for hinged-hinged boundary condition reveals the GDQ method as an accurate and powerful method to solve the governing equations.

1. Introduction

In recent years, study of the Microstructures whose characteristic sizes (thickness, diameter, length, etc.) down to the order of micron and sub-micron has widely increased. The experimental results reveal that the obtained material stiffness and strength in order of micron are higher than those of the bulky materials (Fleck, Muller, Ashby, & Hutchinson, 1994; McFarland & Colton, 2005; Stölken & Evans, 1998). This occurrence is explained as size or scale effects. Since, the classical continuum mechanics theories cannot capture the size effects because they do not contain any internal material length scale parameter, some nonclassical continuum theories such as nonlocal, strain gradient and couple stress theory that contain material length scale parameters have been developed to capture the size effect.

The couple stress theory was introduced in 1960s by Mindlin and Tiersten (1962), Toupin (1962), Mindlin (1964) and Koiter (1964) that contains two higher order material length scale parameters beside the two Lame constants to capture size effect. It should be mentioned that this theory has the lowest material length scale parameters than strain gradient and non-

* Corresponding author. Tel.: +98 21 88005677; fax: +98 21 88013029.
E-mail address: Daneshmehr@ut.ac.ir (A.R. Daneshmehr).

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local theory. Using this theory, Anthoine (2000) studied the pure bending of a circular cylinder and Zhou and Li (2001) investigated the static and dynamic torsion of a circular cylindrical micro-bar. Due to the complication in obtaining more than one higher order material length scale parameter for materials by experiments, Yang, Chong, Lam, and Tong (2002) introduced a new additional equilibrium equation to solve this problem. The new additional equilibrium equation is equilibrium of moments of couples that was added to the classical equilibrium equations of forces and moments of forces and developed a modified couple stress theory that contains only one higher order material length scale parameters in addition to the two Lame constants. Because of containing only one additional constant, this theory has been used extensively in the recent years by researchers. Euler–Bernoulli beams are investigated by Park and Gao (2006) and Kong, Zhou, Nie, and Wang (2008). The static bending and free vibration problems of simply supported Timoshenko and Reddy–Levinson beams are studied by Ma, Gao, and Reddy (2008, 2010) respectively. Mohammad-Abadi and Daneshmehr (2014) studied the buckling analysis of Euler–Bernoulli, Timoshenko and Reddy micro beams based on modified couple stress theory for several boundary conditions that they solved the obtained equations by using the GDQ method. Salamat-Talab, Nateghi, and Torabi (2012) studied bending and free vibration of the simply supported FG micro beams using third order shear deformation theory. Asghari, Kahrobaiyan, and Ahmadian (2010) investigated the static bending and free vibration behavior of size dependent nonlinear Timoshenko beam under constant transverse distributed force for hinged-hinged boundary condition and solved the equations by finite difference method. In recent years, many researchers have applied modified couple stress theory to analyze size-dependent behavior of microstructures (Akgöz & Civalek, 2011;Ke, Wang, Yang, & Kitipornchai, 2012; Şimşek & Reddy, 2013;Wang, Xu, & Ni, 2013; Xia, Wang, & Yin, 2010).

It is obvious that composite materials are increasingly used in many engineering structures and a wide variety of application such as tennis racket, aerospace, mechanical specifically automobile engineering structures and civil engineering structures. Besides, in recent years, substantial use of composite materials in the micro and nano scale in Micro Electro Mechanical System has been noticeable. One can mention microrobots, microsensors, and microactuators as the application of these materials. For instance, due to the better fracture toughness and fatigue properties of composite materials than semiconductors, and higher stiffness to weight ratios of composite materials than most metals, Wood, Avadhanula, Menon, and Fearing (2003) studied the micromechanical flying insect (MFI) made of composite materials. Eigoli and Vossoughi (2010) investigated the motion of a stick–slip microrobot in which each leg is a composite beam that made of piezo-layer and metal base. Ashrafi, Hubert, and Vengallatore (2006) considered the utility of composites as structural materials for designing the microactuators in micro electro mechanical systems. Giedd, Curry, Durham, and Dobson (2003) developed a new class of biosensors made from composite based micro electro mechanical systems.

Due to the using of composite materials in order of micron and sub-micron and also the existence of the micro-scale inhomogeneities and defects such as fibers, impurities and micro-cracks in a laminated composite structure, the classical theory cannot describe the behavior of composite beams especially when the micro-scale phenomena dominates. Therefore, accurate characterization of these composite beams in micron scale is urgent and vital for the reliable and optimal design. So, some researchers have motivated to develop micro composite beams based on modified couple stress theory. Chen, Li, and Xu (2011) proposed a new model for composite laminated beam and studied bending analysis of simply supported composite laminated beams with first order shear deformation and solved obtained equation analytically. Based on proposed modified couple stress theory, Wanji, Chen, and Sze (2012) studied the bending of simply supported composite laminated Reddy beam based on modified couple stress theory by defining the new curvature tensor and solved the obtained equations analytically. Chen and Li (2013) investigated the free vibration of simply supported composite laminated Timoshenko beam based on this theory. Roque, Fidalgo, Ferreira, and Reddy (2012) studied the bending of simply supported composite laminated Timoshenko beam and solved the governed equations analytically and numerically. Mohammad Abadi and Daneshmehr (2014) developed the buckling analysis of Euler–Bernoulli and Timoshenko beams for hinged–hinged boundary condition and solved the governing equations analytically. Using mentioned theory, Chen, Xu, and Li (2012) examined the bending of composite laminated Reddy plate.

Because of the scarcity of vibration analysis of micro composite laminated beams using third order shear deformation theory (Reddy beam model) and GDQ method and considering the effect of different boundary conditions, the authors motivated to study this subject. So, in this paper, vibration analysis of composite laminated beams based on modified couple stress theory and using different beam models i.e. composite laminated Euler–Bernoulli beam (CLEBB), Timoshenko beam (CLTB) and Reddy beam (CLRB) models is developed for the first time. It should be mentioned, in the previous study for composite laminated beams using the Reddy beam model (Wanji et al., 2012), the curvature χ_{zy}, i.e. $\partial\theta_y/\partial z$ is neglected and the couple stress-curvature relation for this model is not comprehensive. Therefore this is the first study that considers this term and presents a comprehensive couple stress-curvature relation and uses third order shear deformation theory, GDQ method and effect of different boundary conditions for dynamic analysis of micro composite laminated beams.

Finally, to present the effect of boundary conditions, three boundary conditions i.e. hinged–hinged (h–h), hinged–clamped (h–c) and clamped–clamped (c–c) are considered and to investigate the effect of lamination on the vibration of micro composite laminated beams, four types of lamination i.e. [0, 0, 0], [0, 90, 0], [90, 0, 90] and [90, 90, 90] are investigated. Using Hamilton's principle, the governing equations and boundary conditions of micro composite laminated beams is obtained. The governing equations of micro composite laminated beams for dynamic analysis are solved numerically. So by using generalized differential quadrature (GDQ) method, natural frequencies of these micro composite laminated beams are obtained. Also, in order to assess the quality of solutions produced by GDQ method, the obtained differential equations are analytically solved for hinged-hinged boundary condition. Comparison between results