International Journal of Engineering Science 87 (2015) 103–109

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Extension of the elasticity-conductivity cross-property connections to impedance of plane microcracked structural elements

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ARTICLE INFO

Article history:
Received 3 October 2014
Received in revised form 21 October 2014
Accepted 1 November 2014
Available online 12 December 2014

Keywords:
Cracks
Resistivity
Impedance
Real-part
Cross-property connection

ABSTRACT

The paper focuses on empirical extension of the cross-property connection between changes in elasto-static compliance and electrical resistance associated with plane micro-cracking in structural elements to changes in electrical impedance. Theoretical underlying principles of impedance are discussed and experimental measurements are taken to compare AC impedance, DC resistance and changes in resistivity predicted by the cross-property connection before and after subjecting test specimens to an edge cut. If the reactive components of impedance are sufficiently isolated, experimental results show a strong correlation between changes in real-part resistive impedance, directly extending the cross-property connection to impedance.

1. Introduction

Connections between changes in different physical properties produced by defects and inhomogeneities were first obtained by Bristow (1960) who derived an explicit connection between the effective electrical conductivity and effective elastic moduli of a solid with multiple randomly-oriented microcracks and verified the connection experimentally. Renewed interest in cross-property connections started in 1980s when Milton (1984) used the minimum potential energy principle to derive an inequality connecting the effective bulk modulus and electric conductivity of a two-phase isotropic composite. His result was further advanced by Berryman and Milton (1988) where cross-property bounds were established using three-point Beran bounds for conductivity (Beran, 1965) and elastic constants (Beran & Molyneux, 1966). Gibiansky and Torquato (1995), Gibiansky and Torquato (1996a), Gibiansky and Torquato (1996b) narrowed the bounds under additional assumptions on the composite micro-geometry and properties of constituents. Explicit cross-property connections have been established by Sevostianov and Kachanov (2002). Their connections are approximate but comparison with experimental data shows good agreement (Sevostianov & Kachanov, 2009).

Cross-property connections interrelate volume average properties. Therefore, strength reduction due to cracks cannot generally be evaluated via changes in the effective elastic compliance. Strength and toughness reduction due to cracks and defects are controlled by local stress fields rather than overall material properties. Sevostianov and Kachanov (2010) proposed an approach utilizing measurement of local changes in electrical conductivities to evaluate fracture-related properties. This approach was experimentally verified by the present authors (Armstrong & Sevostianov, 2014) who developed a

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http://dx.doi.org/10.1016/j.ijengsci.2014.11.004

methodology to reconstruct characteristics of the microstructure from resistance measurements and evaluated the corresponding strength reduction.

All mentioned works on cross-property connections – in explicit form or in the form of bounds, theoretical or experimental – operate with the static properties of materials: elasto-static compliances or stiffnesses are expressed in terms of direct current (DC) electrical conductivities. Alternatively, alternating current (AC) may provide additional information on microstructure or, possibly, increase the sensitivity of experimental data to surface defects (skin depth effect). In the present work we explore the connection between changes in AC and DC characteristics of a material produced by cracks. An analytical relationship for AC cross-property connections is not available in literature to the best of our knowledge. However, by considering electrical impedance in the complex plane with imaginary and real components, simple changes in resistance affect only the real-part of impedance. It is therefore expected that changes in resistivity associated with a crack are directly correlated with changes in the real part of impedance for relatively low frequencies. We provide experimental evidence of this linkage between changes in resistivity associated with cracks to changes in impedance, thereby extending the cross-property connection between compliance and conductivity (resistivity) to impedance.

2. Background

The cross-property connection between changes in Young's modulus in ith direction E_i and electrical conductivity k_i in the same direction due to the presence of multiple microcracks of any (generally non-random) oriented distribution has the following form (Sevostianov & Kachanov, 2009):

$$\frac{E_0 - E_i}{E_i} = \frac{4(1-\nu_0^2)}{2-\nu_0}\frac{k_0 - k_i}{k_i} \equiv C\frac{k_0 - k_i}{k_i}, \quad i = 1,\ 2,\ 3 \tag{1}$$

where E_0, ν_0, and k_0 are Young's modulus, Poisson's ratio and electrical conductivity of the virgin material. Rewritten in terms of the normal compliances, $S_i = 1/E_i$ and resistivities, $r_i = 1/k_i$, this relationship takes the form:

$$(\Delta S_i) = (CS_0/r_0)\Delta r \tag{2}$$

where the change in elastic compliance is related to the change in electrical resistivity, Δr, and the bulk material properties S_0 and r_0. In turn, the change in material resistivity is related to the change in measurable DC electrical resistance by:

$$\Delta r = \Delta R(A/L) \tag{3}$$

where A is the cross-sectional area and L is the length of the specimen. When the specimen is instead subjected to AC, this relationship becomes more complex due to time-variant wave nature of the current and the changing electric and magnetic fields as described by Maxwell's equations. As a result, reactance due to both capacitance and inductance enters the relationship so that complex impedance must be considered. In practice, resistance is measured indirectly from the voltage drop V it creates, based on Ohm's Law (simplified) $V = IR$ where I is current. If the applied current is time variant, the voltage is time variant and impedance, Z, replaces simple resistance in Ohm's Law. As a result, impedance in Ohm's Law is defined by the time variant relationship between voltage and current:

$$Z = V(t)/I(t) \tag{4}$$

The changing current within conductive elements induces electric and magnetic fields, so simple wires have self- and mutual-inductance while parasitic capacitance also exists between elements. The constitutive equations for inductance L and capacitance C must now be considered for Ohm's Law to remain valid:

$$I(t) = CdV/dt; \quad V(t) = LdI/dt \tag{5a \& 5b}$$

When these equations are solved in the context of Ohm's Law, impedance takes on a complex term where reactance from both inductance and capacitance is captured in the imaginary component, with simple resistance belonging to the real part. Plotted in the complex plane, impedance is depicted in Fig. 1 where inductance is positive and capacitance is negative on the imaginary axis and resistance is measured on the real axis:

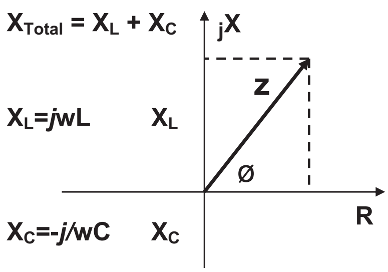

Fig. 1. Reactive (X) and resistive (R) components of impedance (Z) in the complex plane.