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On comparison of thinning fluids used for hydraulic fracturing

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ABSTRACT

The paper aims to answer the question: if and how non-Newtonian fluids may be compared in their mechanical action when used for hydraulic fracturing? We give an answer for thinning fluids by (i) suggesting an appropriate definition of fluid equivalence, and (ii) employing this definition in the analysis of the solution for a fracturing fluid with the power rheological law. The definition accepted in the paper is: two fluids are equivalent in their hydrofracturing action if they produce fractures of the same length at a given reference (treatment) time under the same pumping rate. The solution in self-similar variables, serving for the comparison, is actually independent on fluid behavior index. It implies that for thinning fluids, equivalent in the sense of the definition accepted, the differences in the evolution of main quantities (fracture length, speed, opening, net pressure) are insignificant within the range of time from 10 s to 27 h. It is shown that, at most, the differences may serve to have some quantity greater (less) at time notably less or greater than the reference time. Neglecting the differences, we obtain the equation, which translates the equivalence of thinning fluids in terms of their fracturing action into the equivalence in terms of their rheology. The equation defines the reference strain rate and, consequently, the apparent viscosity, which is the basic value used for fracture design. We conclude that when compared fluids are equivalent in accordance with the equation obtained, the further choice between them is to be made primarily from economic, technological, safety and environmental considerations.

1. Introduction

Hydraulic fracturing is widely used for increasing production of oil, gas and thermal reservoirs. Since the pioneering works by Khristianovich and Zheltov (1955), Perkins and Kern (1961), Geertsma and de Klerk (1969) and Nordgren (1972), it has been a subject of numerous investigations (see, e.g. reviews in the papers by Adachi & Detournay, 2002; Adachi, Siebrits, Pierce, & Desroches 2007; Garagash, 2006; Hu & Garagash, 2010; Linkov, 2012; Mishuris, Wrobel, & Linkov, 2012). Theoretical investigations concerned mostly with asymptotics near the fluid front and regimes of the fracture propagation (e.g., Descroches et al., 1994; Lenoach, 1995; Garagash, Detournay, & Adachi, 2011; Kovalyshen & Detournay, 2009; Mitchell, Kuske, & Peirce, 2007). Benchmark solutions have been found numerically by Nordgren (1972) for the Perkins-Kern-Nordgren (PKN) model, assuming plane-strain conditions in cross sections parallel to the fracture front, and by Spence and Sharp (1985) for the Khristianovich-Geertsma-de Klerk (KGD) model, assuming plane-strain conditions in cross-sections perpendicular to the front. Analytical solutions have been given by Kemp (1990) for the PKN model (see also Kovalyshen & Detournay, 2009) and by Linkov (2012) for both the PKN and KGD models. All these benchmark solutions refer to the case of a Newtonian fluid.

As a rule, fluids used for fracturing are non-Newtonian (see, e.g. Ben-Naceur, 1989; Cameron & Prud'homme, 1989; Economides & Nolte, 2000). The comprehensive review on the properties and considerations used for choices of fracturing

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fluids are given in the key-note lecture by Montgomery (2013). The practical conclusions summarized in the lecture will serve us for further discussion. In particular, since "the fluid viscosity is the major fluid related parameter for fracture design" (Montgomery, 2013, p. 4), it is important to properly account for the non-linear dependence between the shear stress τ and the shear strain rate $\dot{\gamma}$. For engineering purposes, a rough estimation is used in the form of the so-called apparent viscosity μ_a, which is the ratio of shear stress to shear rate at a fixed value a reference shear rate $\dot{\gamma}_r$. The reference value $\dot{\gamma}_r$ is taken quite arbitrary and it varies from tens to hundreds and even thousands. It is desirable to diminish uncertainty by analyzing benchmark solutions for non-Newtonian fluids.

To the author's knowledge, the first solution, accounting for non-linear behavior of thinning fluids, commonly used in practice, was given by Adachi and Detournay (2002) under the assumption of zero fracture toughness. It was extended to the case of non-zero toughness by Garagash (2006). The authors solved numerically the problem for the KGD model. They provided clear pictures of evolution of major quantities (fracture length, speed, opening, pressure) in time for a thinning fluid with prescribed behavior n ($0 < n < 1$) and consistency M indices. However, when using the numerical solutions for *comparison* of various thinning fluids with different behavior indices, the mentioned problem of prescribing a proper value of the reference shear rate arises again. The authors accepted the value $\dot{\gamma}_r$ = 50 1/s (Adachi & Detournay, 2002, p. 594; Garagash, 2006, p. 1464). On whole, this value agrees with the order of shear rates typically expected in fracture; still, as stated by Montgomery (2013, p. 19), "for some soft rock treatments the shear rate may be much lower than this, and in some hard rock treatments, the shear rate may be much greater". Therefore, it is reasonable to account for the influence of rock rigidity and to exclude ambiguity when prescribing the reference shear rate.

The present paper aims to reach this goal by using an analytical rather than numerical solution of the problem for thinning fluids. To this end, we introduce the definition of fluid equivalence in terms of hydrofracturing action and employ it in the analytical solution obtained recently (Linkov, 2013). It appears that the solution in self-similar variables is practically the same in the limiting cases of perfectly plastic ($n = 0$) and Newtonian ($n = 1$) fluids. By continuity, this implies that the self-similar quantities for thinning fluids ($0 < n < 1$) are practically the same for any thinning fluid. This serves us to obtain an equation, which translates the equivalence in terms of hydraulic fracturing effect into the equivalence in terms of the rheological properties. Specifically, it defines the needed shear rate and the apparent viscosity used by engineers for fracture design.

The structure of the paper is as follows. In Section 2, we briefly present the modified formulation and the analytical solution of the PKN problem for the power law fluid. It is stated that the solutions in self-similar variables are actually independent on rheological properties of a thinning fluid. This conclusion is employed in Section 3 in combination with the suggested definition of equivalence of thinning fluids in their hydrofracturing action. We obtain the formula for the reference shear rate, which accounts for the rock elastic properties and the pumping rate. Section 3 contains also the study of the evolution of major dimensional quantities in time. It appears that the differences are insignificant at the range of time from 10 s to 27 h. Besides, it is stated that the results obtained are true when considering the KGD model. In Section 4, we derive equations for the reference shear rate and apparent viscosity. A brief summary is given in Section 5.

2. Modified formulation and self-similar solution of PKN problem

2.1. Modified formulation of PKN problem

We consider the geometrical scheme of the PKN model (Fig. 1), for which plane-strain conditions occur in cross sections parallel to the fracture front. Then the elasticity equations yield the proportionality of the fracture opening w to the net pressure p (Nordgren, 1972):

$$p = k_s w, \tag{1}$$

where $k_s = (2/\pi h)E/(1 - \nu^2)$, E is the elasticity modulus, ν is the Poisson's ratio, h is the fracture height. The hydraulic fracture is driven by a fluid with the power-law rheology:

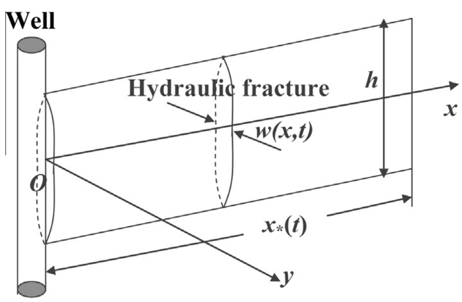

Fig. 1. Sketch of the Nordgren problem.