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Instabilities of an electroelastic plate

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ABSTRACT

In this paper the theory of nonlinear electroelasticity is used to examine diffuse modes of instability for two problems involving a thin dielectric plate subject to large deformations. The analysis is based on the equations governing linearized incremental deformations and electric displacements and accompanying boundary conditions superimposed on a known finitely deformed configuration in the presence of a known electric field (the underlying configuration). For each problem the underlying deformation is taken to correspond to an equibiaxial stretch with the electric field normal to the major faces of the plate. In the first problem the electric field is generated by equal and opposite charges applied to flexible electrodes on the major surfaces, while in the second problem there are no electrodes and the field is applied externally.

Numerical results are obtained in respect of a simple model of an electroelastic material by way of illustration. The critical stretch corresponding to loss of stability of the uniform underlying configuration is obtained as a function of a dimensionless measure of the initial thickness of the plate for a series of values of the electric (displacement) field and of the parameters included in the material model. For the first problem, the results obtained for the critical stretch are compared with the results based on the so-called Hessian approach, which provides only a limited analysis of stability. It neither takes account of the plate thickness nor allows for non-homogeneous deformations, and significant differences in the predictions of the two methods are identified.

1. Introduction

Considerable interest has developed in the last few years in the possibilities for using dielectric elastomers as electromechanical transducers such as sensors and actuators, as has been nicely summarized in the collection of papers edited by Carpi, De Rossi, Kornbluh, Pelrine, and Sommer-Larsen (2008) and the review by O'Halloran, O'Malley, and McHugh (2008) and illustrated by many papers containing experimental data and theoretical results, including, for example, Wissler and Mazza (2007), Plante and Dubowsky (2007), Brochu and Pei (2010), Keplinger, Li, Baumgartner, Suo, and Bauer (2012) and Li et al. (2013).

A typical actuator, for example, consists of a thin film of electroactive elastomer on the major surfaces of which are coated two flexible electrodes. Application of a potential difference between the electrodes causes thinning of the film and its lateral expansion (Blok & LeGrand, 1969; Pelrine, Kornbluh, Pei, & Joseph, 1998) along with that of the electrodes, thus converting electrical energy into mechanical action and activation. Pelrine, Kornbluh, Pei, and Joseph (2000), using thin films of silicone and acrylic elastomers, recorded actuation strains well above 100%. However, the thinning of the film can become unstable, a phenomenon known as 'pull-in instability'.

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In an early experimental paper Stark and Garton (1955) attributed rapid changes in the electrical strength of rubber-like sheets to an unstable reduction in their thickness when the applied electric field exceeded a certain critical value. To describe the equilibrium state, they proposed a nonlinear relation connecting the electric potential difference and the mechanical restoring force. Blok and LeGrand (1969) showed experimentally that a thin polymeric plate subject to a uniform electric field in the direction normal to the top and bottom faces and with magnitude close to the critical value is susceptible to non-uniform electromechanical thinning. They interpreted this as being caused by microscopic imperfections in the material that experience higher than average fields, resulting in confined indentations. The electric boundary conditions require that on the indentation surface the direction of the applied field changes and the intensity of the electric field ceases to be uniform, increases locally, and induces electromechanical instability, inhomogeneous deformations, such as wrinkling (Plante & Dubowsky, 2006), and electrical breakdown.

Such instability phenomena pose problems for the design of actuator devices and careful modelling and analysis is therefore required. Providing a general method for analyzing diffuse modes of instability is the main purpose of the present paper.

The experimental results of Plante and Dubowsky (2006) have motivated much of the recent theoretical developments. For example, Zhou, Hong, Zhao, Zhang, and Suo (2008) used numerical methods to simulate the initial uniform thinning of an electroelastic plate and, following pull-in instability, the coexistence of homogenous and non-homogeneous states, while De Tommasi, Puglisi, Saccomandi, and Zurlo (2010) developed a simplified model to account for the initial uniform thinning followed by non-homogeneous deformation of thin electroelastic films. In the paper by De Tommasi, Puglisi, and Zurlo (2011) an approach based on tension field theory was used to determine the onset of compression induced wrinkling. More recently, the same authors employed an energy approach to determine the existence of non-homogeneous equilibrium configurations in thin films (De Tommasi, Puglisi, & Zurlo, 2013a, 2013b). See also the analysis in the recent paper of Díaz-Calleja, Llovera-Segovia, Jorge Dominguez, Carsí Rosique, and Quijano Lopez (2013), which was concerned, in particular, with wrinkling instabilities.

Puglisi and Zurlo (2012) have evaluated the effect of thickness imperfections on electromechanical instabilities of a dielectric elastomer capacitor, while Zurlo (2013) proposed a simple constitutive dependence on the second gradient of the deformation and analyzed non-homogeneous deformations in order to estimate the onset of pull-in instability in electroelastic films and found that the resulting non-local effects can significantly decrease the instability threshold. A finite element method was used by Park, Suo, Zhou, and Klein (2012) to study inhomogeneous deformations in dielectric elastomers subject to general electrostatic loading and the instability of multilayered dielectric composites has been investigated by Bertoldi and Gei (2011) and Rudykh and deBotton (2011) using the theory of linearized incremental deformations and electric fields superimposed on a known underlying deformation and electric field. In a very recent paper, which has some features in common with the present work, Gei, Colonelli, and Springhetti (2013) investigated the effect of a homogeneous in-plane extension on the stability of an electroelastic plate. Specifically, for a pre-stressed plate pull-in and localized instabilities were discussed, including diffuse mode instabilities based on the theory of Dorfmann and Ogden (2010a) and Bertoldi and Gei (2011), and 'shear band' type instabilities.

Theoretical predictions of instabilities in electroelastic plates (more specifically thin plates or films) are frequently restricted to deformations where the top and bottom faces remain perfectly plane. Use of the so-called Hessian approach restricts attention to homogeneous deformations and does not involve the thickness of the film. This approach has been used by, for example, Zhao, Hong, and Suo (2007), Zhao and Suo (2007), Moscardo, Zhao, Suo, and Lapusta (2008), Díaz-Calleja, Sanchis, and Riande (2009a), Leng, Liu, Liu, Yu, and Sun (2009), Díaz-Calleja, Sanchis, and Riande (2009b), Liu, Liu, Sun, and Leng (2010), Suo (2010), Zhu, Stoyanov, Kofod, and Suo (2010), Xu, Mueller, Klassen, and Gross (2010), Zhao, Koh, and Suo (2011) and Hong (2011), and on a similar basis Jiménez and McMeeking (2013) have recently evaluated the influence of a deformation dependent permittivity tensor on the electromechanical instability of thin films.

The objective of the present paper is to investigate the stability of an electroelastic plate with finite thickness that takes account of the plate thickness and non-homogeneous deformations. We focus on the equations governing the linearized incremental deformation and electric displacement field superimposed on a known finitely deformed configuration in the presence of an electric field. We then specialize the equations to two specific problems. First (Problem 1), we consider a dielectric elastic material forming a cuboidal plate whose thickness is small compared with its lateral dimensions and has flexible electrodes attached to its (parallel) major surfaces. A uniform electric field is generated through the material by (equal and opposite) uniform surface charges on the flexible electrodes by application of a potential difference between the electrodes. In the second problem (Problem 2), the stability of the same plate, but without the electrodes, is analyzed for the case in which an external electric field is applied normal to the major surfaces. Particular attention is paid to the incremental electric boundary conditions in each problem.

The paper is organized as follows. First, in Section 2, we provide a general overview of the basic equations of nonlinear electroelasticity and then specialize the theory to isotropic materials based on the formulation of Dorfmann and Ogden (2005). Then, following Dorfmann and Ogden (2010a), we summarize the equations governing linearized incremental deformations and electric displacements superimposed on a known finitely deformed configuration in Section 3. In particular, we give formulas for the incremental constitutive equations and the incremental electric and mechanical boundary conditions. In Section 4 the general theory is then applied in the solution of the two problems mentioned in the previous paragraph. In each case the analysis is presented for a general isotropic electroelastic constitutive law. The underlying deformation is then taken to correspond to an equibiaxial stretch, and for purposes of illustration numerical results are obtained for a simple model of an electroelastic material. The critical stretch corresponding to loss of stability is obtained as a function of a