International Journal of Engineering Science 78 (2014) 48–60

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Modeling collagen recruitment in hyperelastic bio-material models with statistical distribution of the fiber orientation

Alessio Gizzi [a,*], Marcello Vasta [b], Anna Pandolfi [c]

[a] University Campus Bio-Medico of Rome, Engineering Faculty, via A. del Portillo 21, 00128 Rome, Italy
[b] Università di Chieti-Pescara, Dipartimento INGEO, Viale Pindaro 42, Pescara, Italy
[c] Politecnico di Milano, Dipartimento di Ingegneria Civile ed Ambientale, Piazza Leonardo da Vinci 32, Milano, Italy

ARTICLE INFO

Article history:
Received 26 December 2013
Received in revised form 28 January 2014
Accepted 8 February 2014
Available online 3 March 2014

Keywords:
Statistical fiber distribution
Fiber recruitment
Hyperelasticity
Anisotropy second order approximation

ABSTRACT

Gradual fiber recruitment is one of the stiffening mechanisms observed in collagen reinforced biological tissues. Given the natural statistical distribution of the fiber orientation in biological materials, in agreement with experimental findings it is reasonable to assume a stochastic nature of the fiber recruitment mechanism. In the present study, we consider the presence of a stochastic recruitment mechanism in a hyperelastic fiber reinforced material model characterized by statistical distributions of the fiber orientation. The material model is based on a second order approximation of the strain energy density, considered as a function of the fourth pseudo-invariant $\bar{I}_4$, and on the multiplicative decomposition of the deformation gradient and, consequently, of the stretch. For a planar distribution of the fiber orientation, we choose an exponential analytical expression of the strain energy density and derive the stress and stiffness tensors. The mechanical behavior of the model is assessed, through uniaxial tests, by distinguishing the mean and the variance contributions of $\bar{I}_4$ to the model is validated against experimental data.

1. Introduction

Fiber reinforced bio-materials show strongly nonlinear and anisotropic responses (Destrade et al., 2009; Merodio and Ogden, 2005). Many biological tissues, in particular, are characterized by a functional micro-architecture of collagen fibers and are modeled conveniently as an underlying isotropic matrix embedded with one or more sets of fibers (Fereidoonnezhad et al., 2013). At small strains the dominant response is provided by the isotropic component, while for larger strains the collagen fibers, initially crimped, gradually unfurl and begin bearing some loading (Roach and Burton, 1957). The contribution of the uncrimped fibers becomes dominant for large strains.

Most of material models commonly used in applications assume that the contribution of fibers, although marginally, manifests even at low strains (Gasser et al., 2006; Holzapfel et al., 2000; Pandolfi and Vasta, 2012). There are experimental examples, though, of activation of the fibers at a particular, finite, strain threshold. Abrupt fiber recruitment has been considered in material modeling (Li and Robertson, 2009; Watton et al., 2012; Wulandana and Robertson, 2005), but the direct measurement of the process has been posing challenges to experimentalists. Recent studies proposed to use multi-photon microscopy (MPM) combined with biomechanical devices to visualize the collagen structure in segments of biological tissue

* Corresponding author. Tel.: +39 06225419660.
E-mail addresses: a.gizzi@unicampus.it (A. Gizzi), mvasta@unich.it (M. Vasta), anna.pandolfi@polimi.it (A. Pandolfi).

http://dx.doi.org/10.1016/j.ijengsci.2014.02.008
0020-7225/

at different strains (Megens et al., 2007; Zoumi et al., 2004). An accurate experimental program on carotid arteries where non destructive uniaxial tests joined with MPM provided a good collection of stretched collagen fiber and elastin images has been recently discussed in Hill et al. (2012).

As far as the theoretical aspects are concerned, several years ago Lanir (1979, 1983) formulated a three-dimensional structural model of planar collagenous tissues—e.g. skin—accounting for the distribution of collagen fiber orientation and for fiber recruitment. Subsequently, Sacks (2000) incorporated directly in Lanir's model the distribution of the fiber orientation obtained from small angle light scattering, and characterized the recruitment probability distribution function (PDF) with parameters obtained from a least-squares fit of experimental data. More recently collagen fiber recruitment defined by a PDF, but assumed to initiate at an infinitesimal strain, has been quantified in rabbit carotid arteries using staining and fixation with confocal microscopy (Roy et al., 2010).

On the basis of their own experimental results, to describe gradual recruitment at finite strains Hill et al. (2012) proposed an alternative form of the strain energy function, characterized by stretches defined on a PDF. The mechanical parameters feeding the material model were identified directly from experimental measurements.

Some biological tissues show a relatively thin two-dimensional structure, resulting from the particular interlacing architecture of reinforcing collagen fibers. Recently, the mathematical characterization of such tissues has seen important advances (Soldatos, 2009). Among others, we can mention the spatial distribution of the fiber orientation (Alastrué et al., 2006; Federico and Gasser, 2010; Gasser et al., 2006; Kroon and Holzapfel, 2008; Pandolfi and Vasta, 2012; Pinsky et al., 2005; Raghupathy and Barocas, 2009; Roy et al., 2010; Vasta et al., 2013; Wang et al., 2012).

In the present contribution we consider a class of material models characterized by a spatial distribution of the orientation of the fibers, adopting an approximated formulation of the strain energy density. We start from the Taylor expansion of the strain energy about the average fourth invariant $\bar{I}_4^*$ of the distribution (Pandolfi and Vasta, 2012), truncated at the quadratic terms. Then, the stochastic material model, referred to as *second order, or variance, approximation*, is enriched with a collagen recruitment mechanism (Hill et al., 2012), described with a recruitment function likewise expanded about $\bar{I}_4^*$ up to the quadratic terms. We derive the explicit expressions of the stress and elasticity tensors of the proposed material model. The new stochastic model is compared with previous material models and validated against experimental results.

We show that the highly nonlinear behavior of the recruitment model is well described by the proposed second order formulation of a strain energy with a von Mises distribution of the fiber orientation. Numerical simulations of uniaxial tests show that the approximation provided by the second order or variance approach is more accurate than the approximation based on the first order, or mean, formulation. Upon experimental data set tuning, the present theoretical formulation allows for the parametric characterization of the mean and variance contributions to the average stress and tangent stiffness tensors.

It is important to emphasize that the present approach is slightly different from others currently discussed in the literature. In particular, we note that the expression *second order* used here to denote the approximation of a statistical distribution of fibers should not be confused with the second order terms of the Landau and Lifshitz expansion of isotropic strain energy densities (Landau and Lifshitz, 1986) described, e. g., in Destrade et al. (2010). Additionally, we remark that the fiber families here considered are characterized by a distribution of the spatial—or planar—orientation and they do not fall in the class of transversally isotropic materials, except for the case of fully aligned fibers. However, for the latter case, it is not worth to use the present model. According to Spencer, transversally isotropic hyperelastic fiber reinforced materials must be described in terms of the Cauchy strain invariants $\bar{I}_1$ and $\bar{I}_2$ and of two pseudo invariants $\bar{I}_4$ and $\bar{I}_5$ that include the orientation of the fiber. Recently, it has been pointed out that the description of fiber reinforced transversally isotropic materials cannot be done in terms of only a pair of strain invariants (e. g., $\bar{I}_1$ and $\bar{I}_4$). In fact, this assumption introduces a kinematic constraint between the stretch components that cannot be satisfied in simple tension tests (Destrade et al., 2013; Murphy, 2013; Pucci and Saccomandi, 2014). Our approach, though, does not face this issue, since the description of the isotropic part of the hyperelastic strain energy is done in terms of a Mooney–Rivlin model, which depends on both $\bar{I}_1$ and $\bar{I}_2$, while the anisotropic part accounts for a distributed, not aligned, set of fibers that is described by means of average and variance measures.

The outline of the paper is as follows. In Section 2 we introduce the general hyperelastic formulation and the statistical distribution of fibers in terms of both first and second order approximations. In Section 3 we specialize the general three-dimensional formulation to a planar fiber distribution. In Section 4 we introduce material models able to describe the fiber recruitment process. The numerical response and parameter tuning upon experimental uniaxial tests are reported in Section 5, whereas conclusions and future perspectives are drawn in Section 6.

2. Fully three-dimensional fiber orientation distribution

We are concerned with a fully hyperelastic approach, and introduce a strain energy function that describes the reversible behavior of a fibrous material characterized by reinforcing fibers embedded in an isotropic matrix. We make the assumption that the strain energy density splits into the sum of volumetric, matrix, and fiber contributions in the standard form:

$$\Psi(\mathbf{C},\mathbf{a}) = \Psi_{\text{volumetric}}(J) + \Psi_{\text{matrix}}(\overline{\mathbf{C}}) + \Psi_{\text{fiber}}(\overline{\mathbf{C}},\mathbf{a}),$$

where $\overline{\mathbf{C}} = J^{-2/3}\mathbf{C}$ is the isochoric part of the Cauchy–Green deformation tensor $\mathbf{C} = \mathbf{F}^T\mathbf{F}, \mathbf{F}$ the deformation gradient, $J = \det(\mathbf{F})$ the volumetric deformation, and $\mathbf{a}$ a unit vector describing the orientation of the fiber distribution in the stress free state. The expressions of $\Psi_{\text{volumetric}}(J)$ and $\Psi_{\text{matrix}}(\overline{\mathbf{C}})$ will be specified later in the application section, since we are