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Wave scattering by a thin vertical barrier in a two-layer fluid

Harpreet Dhillon [a], Sudeshna Banerjea [a], B.N. Mandal [b,*]

[a] Department of Mathematics, Jadavpur University, Kolkata 700032, India
[b] Physics and Applied Mathematics Unit, Indian Statistical Institute, 203 B.T. Road, Kolkata 700108, India

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ABSTRACT

The present paper is concerned with scattering of surface and interface waves by a vertical plate in a fluid consisting of a layer of finite depth bounded above by a free surface and below by an infinite layer of fluid of density greater than the upper layer. For such a situation time-harmonic waves can propagate with two different wavenumbers K and v $(> K)$ along the free surface and the interface respectively. The problems are formulated in terms of hypersingular integral equations by suitable applications of Green's integral theorem in terms of difference of potential function across the barrier. These integral equations are solved by a collocation method using a finite series involving Chebyshev polynomials. Reflection and transmission coefficients for incident waves of wavenumbers K and v are computed numerically and depicted graphically in a number of figures for various values of different parameters. The energy identities are used as a partial check on the correctness of the numerical results.

1. Introduction

Stokes (1847) first investigated propagation of waves in na two-layer fluid assuming linear theory. In the classical treatise by Lamb (1932, Section 231), it was shown that in a two-layer fluid with a free surface there exists two possible linear wave systems at a given frequency each with a different wavenumber, the waves with lower wavenumber (or mode) propagates along the free surface while those with higher wavenumber propagates along the interface. When a wave train of a particular mode encounters an obstacle, it is partially reflected into waves of both modes and also partially transmitted similarly. Thus there is a transfer of energy from surface wave to interface wave and vice-versa. This makes the study of wave scattering problems in a two-layer fluid interesting. Linton and McIver (1995) developed the general theory for two-dimensional motion in a two-layer fluid in which the lower layer of heavier density extends infinitely downwards and the upper layer of lower density has a free surface. They investigated the problem of two-dimensional wave scattering by a long horizontal circular cylinder submerged in either layer by employing multipole expansion method. This problem arose due to the plan to build submerged pipe bridge across a Norwegian fjord consisting of a layer of fresh water on top of a deep layer of salt water. Linton and Cadby (2002) extended the work of Linton and McIver (1995) to oblique scattering. Manam and Sahoo (2005) obtained analytical solutions for the radiation and scattering of oblique waves by a porous barrier in a two-layer fluid having a free surface. Das and Mandal (2007) extended the problem of Linton and McIver (1995) and Linton and Cadby (2002) where the upper layer has a thin ice-cover modelled as a thin infinite elastic plate. Wave scattering problems involving thin vertical barriers in a two-layer fluid have gained considerable interest in the literature recently. The corresponding problem for a

* Corresponding author. Tel.: +91 3325753034; fax: +91 3325773026.
E-mail addresses: harpreetdhillon1186@gmail.com (H. Dhillon), sbanerjee@math.jdvu.ac.in (S. Banerjea), biren@isical.ac.in (B.N. Mandal).

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single fluid bounded by a free surface wherein the barrier is either fully submerged and extends infinitely downwards or partially immersed or submerged in an infinitely deep water was well studied by Dean (1945), Ursell (1947) and Evans (1970) and many others employing a variety of mathematical techniques. This class of problems in infinitely deep water is among limited problems which admit of closed form solutions. Mandal, Banerjea, and Dolai (1995) considered two superposed fluids wherein the upper fluid extends infinitely upwards and the lower fluid extends infinitely downwards and a thin vertical barrier is submerged in the lower fluid and extends infinitely downwards. Dolai and Mandal (1996) investigated the problem of interface wave diffraction by a thin finite flat plate submerged in the lower fluid of two superposed infinite fluids, the plate being inclined at an arbitrary angle with the vertical. They used a hypersingular integral equation formulation and employed the numerical procedure of Parsons and Martin (1992, 1994) to obtain the reflection and transmission coefficients.

The present paper is concerned with scattering of surface and interface waves by a vertical plate partially immersed in the upper layer of a two-layer fluid, the upper layer being of finite height and having a free surface while the lower layer extending infinitely downwards. By suitable application of Green's integral theorem in the two-fluid region and taking care of the interface conditions in an appropriate manner, the problem is formulated here in terms of a hypersingular integral equation for the difference of velocity potential across the barrier. The hypersingular integral equation is then solved numerically by using a collocation method based on Chebyshev polynomial approximation (Parsons & Martin, 1992, 1994). The solution of this hypersingular integral equation is then utilized to compute the reflection and transmission coefficients. These were obtained numerically and presented graphically for various values of the wavenumber. From the numerical results it is found that reflection and transmission coefficients for both surface waves and interface waves exhibit oscillatory behaviours as the ratio of densities of upper and lower fluid increases. The energy identities are used to check the correctness of the numerical results presented here.

2. Statement of the problem

We consider two-dimensional irrotational, time-harmonic motion in two-layer fluid of which the upper layer is of density ρ_1 and the lower layer is of density $\rho_2(>\rho_1)$. A thin rigid vertical plate described by $x = 0,\ 0 < y < a$, is partially immersed in the upper layer which occupies the region $0 < y < h\ (h > a)$ with $y = 0$ as the mean free surface, y-axis being taken vertically downwards. The lower fluid occupies the region $h < y < \infty$ where $y = h$ is the undisturbed mean interface of the two fluids. Under the usual assumptions of linear theory, the velocity potentials describing fluid motions in the upper and lower fluid regions are

$$\Phi(x,y;t) = Re(\phi(x,y)e^{-i\omega t}),$$
$$\Psi(x,y;t) = Re(\psi(x,y)e^{-i\omega t}),$$

respectively, where ω is the circular frequency. Here, ϕ and ψ satisfy the Laplace equation

$$\nabla^2\phi = 0, \quad 0 < y < h, \tag{2.1}$$
$$\nabla^2\psi = 0, \quad h < y < \infty. \tag{2.2}$$

Linearized boundary conditions at the free surface, plate and at the interface are

$$K\phi + \phi_y = 0, \quad \text{on } y = 0, \tag{2.3}$$
$$\phi_x = 0, \quad \text{on } x = 0,\ 0 < y < a, \tag{2.4}$$
$$\phi_y = \psi_y, \quad \text{on } y = h, \tag{2.5}$$
$$s(K\phi + \phi_y) = K\psi + \psi_y, \quad \text{on } y = h, \tag{2.6}$$

where $s = \frac{\rho_1}{\rho_2}$ and $K = \frac{\omega^2}{g}$, g being the acceleration due to gravity.

Also, the bottom condition is

$$\nabla\psi \to 0, \quad \text{as } y \to \infty. \tag{2.7}$$

In a two-layer fluid, progressive waves are described by

$$\phi = Ae^{\pm iux}[(K\sigma - u)e^{2uh}e^{-uy} + (K - u)e^{uy}], \tag{2.8}$$
$$\psi = Ae^{\pm iux}K(\sigma - 1)e^{2uh}e^{-uy}, \tag{2.9}$$

where u satisfies the dispersion relation

$$(u - K)[K(\sigma + e^{-2uh}) - u(1 - e^{-2uh})] = 0. \tag{2.10}$$

with $\sigma = \frac{1+s}{1-s}$. It follows that $u = K$ and $u = v$ where

$$(K + v)e^{-vh} + (K\sigma - v)e^{vh} = 0. \tag{2.11}$$