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Variational principles and size-dependent bounds for piezoelectric inhomogeneous materials with piezoelectric coherent imperfect interfaces

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ABSTRACT

In most practical situations, the interfaces between the constituent phases of composites or inhomogeneous materials are imperfect. The present work aims to establish the bounds for the effective properties of an inhomogeneous material with coherent imperfect interfaces in the setting of piezoelectricity. According to this imperfect interface model, the displacement and the electrical potential are continuous while the traction and the normal electrical displacement are discontinuous across an interface and proportional to the surface gradient of the displacement and the electrical potential, respectively, by the so-called Laplace–Young equation. To achieve this objective, firstly, the classical minimum potential principles of linear piezoelectricity are extended to such inhomogeneous material and to formally setting bounds for their effective piezoelectric properties. Secondly, we consider a transversely isotropic piezoelectric composite consisting of a matrix reinforced by cylindrical inhomogeneities via linearly piezoelectric coherent imperfect interface which is subjected to the uniform anti-plane mechanical load and in-plane electrical load boundary conditions. By taking simple trial strain and electrical displacement or simple trial traction and electrical potential couple fields, the first-order upper and lower bounds are explicitly derived for the corresponding elastic, piezoelectric and dielectric moduli of such composite by using the established variational principles. Finally, numerical results of the obtained bounds are provided to illustrate their size-dependence.

1. Introduction

The interfaces between the constituent phases of composites or heterogenous media are classically assumed to be perfect. However, in most practical situations, the imperfect interfacial bonding may exist in such materials and greatly affect their overall mechanical or physical behavior. Therefore, the assumption of perfect interface is sometimes not inadequate in the practical situations. In the setting of piezoelectricity, an interface is said to be perfect if the displacement, the traction, the electrical potential and the normal electrical displacement are all continuous across it. Otherwise, this interface is qualified as being imperfect.

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The prediction of the imperfect interface effects on the overall mechanical or physical behavior of various composites or inhomogeneous materials has attracted a lot of attention of researchers in many fields (see, e.g., Chen, Dvorak, & Yu, 2007; Duan, Wang, Huang, & Karihaloo, 2005; Fang, Liu, Jin, & Wen, 2009; Hashin, 1990, 1991; Le-Quang, Bonnet, & He, 2010; Le-Quang, He, & Bonnet, 2011; Nan, Birringer, Clarke, & Gleiter, 1997; Nan, Huang, & Weng, 2001; Steigmann & Ogden, 1997, 1999; Sharma & Ganti, 2004; Wang & Pan, 2010; Wang & Sudak, 2007). In the framework of micromechanical analysis, there are three main approaches which have been adopted in the estimation of the overall behavior of composites in the literature. The first one is the direct prediction of the effective properties. For example, by driving the closed-form expression for the generalized Eshelby's interior and exterior tensor fields, Le-Quang et al. (2010, 2011) estimated the overall thermal-conduction properties of composite with Kapitza's interface thermal resistance and highly conducting imperfect interfaces, Zhong and Meguid (1997) and Duan et al. (2005) predicted the size-dependence overall elastic properties of a composite with elastic spring-type and coherent imperfect interfaces, respectively. By using the generalized self-consistent schemes, Xiao, Xu, and Zhang (2011) gave the size-dependent effective moduli of piezoelectric nanocomposites with piezoelectric coherent imperfect interfaces. Benveniste and Miloh (1986) investigated the effective conductivity of composites with Kapitza's interface thermal resistance interfaces between the constituents on basis of the localization tensors obtained from the solution of an auxiliary inhomogeneity problem of a spherical particle embedded in an infinite reference medium under a remote intensity. Nan et al. (1997), Duan, Yi, Huang, and Wang (2007a, 2007b) proposed the equivalent inhomogeneity method to estimate the overall conductivity and elastic behavior of the composite with special imperfect interfaces, respectively, where the imperfect interface effects were replaced by an equivalent inclusion. The second one involves the exact connections between internal fields and various components of effective moduli. For example, Duan and Karihaloo (2007) and Chen (2008) established the exact size-dependent connections for the composite with special imperfect interfaces in the setting of thermoelasticity and piezoelectricity, respectively.

The third one predicts the overall properties of heterogeneous materials by establishing their upper and lower bounds. For composites with perfect interfaces, these bounds have been successfully obtained for the uncoupled phenomena such as conductivity or elasticity (Hashin & Shtrikman, 1962; Nemat-Nasser, 1999) and for the coupled phenomena such as piezoelectricity (Hori & Nemat-Nasser, 1998; Li & Dunn, 2001). In the case of the imperfect interface involved, Hashin (1992) and Le-Quang and He (2008) firstly generalized the minimum potential and complementary energy principles to the inhomogeneous materials with linear spring type and coherent imperfect interfaces in order to construct bounds for their effective elastic properties, respectively. Variational principles analogous to those of Hashin (1992) and Le-Quang and He (2008) are developed by Lipton (1997) and Lipton and Vernescu (1996) for the effective conductivity of composites with highly conducting imperfect interface and Kapitza's interface thermal resistance. When the interface effects are neglected, the results of Hashin (1992), Le-Quang and He (2008), and Lipton (1997) recover the classical Voigt and Reuss bounds. New principles of comparison type for Kapitza's interface thermal resistance and highly conducting imperfect interface are introduced by Lipton (1997), Lipton and Vernescu (1996), and Wu (2010) which can be reduced those of Hashin and Shtrikman (1962) in the limit of perfect interfacial bonding. Even much efforts in obtaining variational bounds for the effective properties of the composite with imperfect interfaces, to the best of the authors' knowledge, the attention has not been paid to the coupled phenomena.

In this paper, we consider the piezoelectric inhomogeneous media as an illustrated example of establishing the closed form of variational bounds for the coupled phenomena in which the imperfect interfaces are characterized by the linearly piezoelectric coherent interface model. This interface model was first proposed by a piezoelectric extension of Gurtin and Murdoch (1975) on the basis of continuum mechanics and thermodynamics. Then, Benveniste (2006) derived the general elastic interface model by applying the Taylor's expansion technique to replace a layer of small uniform thickness with a zero thickness interface and showed in a physically sound and mathematically rigorous way that the coherent elastic interface can be retrieved from the obtained general one by taking a very stiff interphase. This rigorous approach to derive the special elastic interface models was initiated by Pham-Huy and Sanchez-Palencia (1974) and Sanchez-Palencia (1970) who considered a weakly or highly conducting thin interphase between two neighboring phases and applied mathematical asymptotic analysis to retrieve the special Kapitzas interfacial thermal resistance model and highly conducting interface model. More recently, this approach was also extended by Wang and Pan (2010) to the setting of piezoelectricity and showed that the resulting imperfect interface model corresponds to the coherent interface one if the conductivity and elastic stiffness of the interphase are much higher than the ones of the surrounding bulk phases for plane interfaces, and then extended by Gu (2008) and Gu, Liu, and He (2013) for arbitrary curve interfaces. It is important to observe that the piezoelectric coupling is not present in this case. The present paper has two objectives. First, it aims to extend the classical minimum potential principles of linear piezoelectricity to including the imperfect coherent interface effects. The second purpose is to apply the extended variational principles to establishing bounds for the effective piezoelectric moduli of the composite under consideration. The variational principles obtained in the present paper follow from the works of Hashin (1992) and Le-Quang and He (2008). When the interface effects are neglected, the obtained variational principles recover the classical piezoelectric Voigt and Reuss bounds. It should be emphasized that the present work can be viewed as a generalization of Le-Quang and He (2008) to the setting of piezoelectricity.

The present paper is organized as follows. In the next section, the phase constitutive laws, the coherent interface model and the effective relations in the setting of piezoelectricity are specified. In Section 3, the extended variational principles of potential for the piezoelectric composite with the piezoelectric coherent interfaces are deduced. In Section 4, these extended variational principles are applied to formally build the expressions of the upper and lower bounds for the composite under