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Combined stretching and twisting of a circular thin-walled tube of second gradient plastic material

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ABSTRACT

The objective of this work is to derive an analytical solution for the problem of a second gradient plastic circular thin-walled tube in combined tension and torsion loading conditions. Explicit expressions for the ordinary and higher-order stresses, strain rate and its gradient, and displacement are obtained. For comparison purpose, we also present the analytical solution for the same problem when the thin-walled tube is made of classical (von Mises) plastic metals. The newly micromorphic model based solution reduces to that of the (ordinary) von Mises one when the characteristic length scale approaches zero.

1. Introduction

The recent increasing demand to understand the role played by microstructures on mechanical properties (strength, toughness, ductility etc.) and failure of heterogeneous materials such as metals, ceramics, and aluminum under various external loading conditions has popularized the use of microstructure related constitutive theories, so-called generalized continuum mechanics. The fundamental difference between generalized continuum theories and the classical continuum theories is that the former is a continuum model embedded with microstructures (nonlocal models) aimed at describing the microscopic motion or long range material interaction. The underlying idea of generalized continuum theories is to extend the application of continuum theories, which are macroscopic by essence, to microscopic space and short time scales. There are several of generalized continuum theories. Without being exhaustive, let mention the micromorphic theory of Eringen and Suhubi (1964) and Eringen (1992) where the material body is a continuum set of large number of deformable particles of finite size and inner structure. The micromorphic theory is identical to (i) Mindlin (1964)'s microstructure theory in the context of small strain and slow motion (ii) micropolar theory when the microstructure of the material is assumed to be rigid, see Eringen and Suhubi (1964). When a constant microinertia is assumed, micropolar theory is reduced to the Cosserat and Cosserat (1906)'s theory; when the macromotion of the particle is identical to the micromotion of the structure, the micropolar theory becomes the couple stress theory of Mindlin and Tiersten (1962) and Toupin (1962). New material model parameters unavoidably emerge in generalized continuum theories, which raises the critical question of their determination. To circumvent this problem, reduced models with manageable number of length-scale parameters have been developed. These length scales are usually found to be related to the material heterogeneities such as the maximum aggregate size in concrete materials as shown by Bazant and Pijaudier-Cabot (1989), the mean distance between voids in porous metallic materials as demonstrated by Tvergaard and Needleman (1995, 1997), and the dislocation mean free path in metal plasticity

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as studied by Abu Al-Rub and Faruk (2012). Sometimes, even though the characteristic length scale in the model is simply introduced phenomenologically into the constitutive models, it can *a posteriori* be related to the microstructure of the material. For instance, let mention the work of Tvergaard and Needleman (1997) where the characteristic length scale, introduced by adopting in the constitutive model a convolution integral for the rate of increase of voids, provided insight in the fracture mechanisms in porous metals.

Generalized continuum theories were first introduced to describe elastic materials with microstructures, see for instance Mindlin (1964, 1965) and Germain (1973). Later on, the use of generalized continuum theories has been extended to predict the inelastic behavior of materials, in particular in metal plasticity. Some of the main contributions in this context include, but are not limited to the works of Aifantis (1984, 1995), Bammann and Aifantis (1982),Clayton, McDowell, and Bammann (2006), Solanki and Bammann (2010), Fleck and Hutchinson (1993, 1997) and Fleck, Muller, Ashby, and Hutchinson (1994) where a gradient of the deformation is introduced into the constitutive equations following some *ad hoc* or more refine phenomenological arguments. The generalized continuum theory concept has also been applied to model ductile fracture in porous plastic metals, see for instance the work of Tvergaard and Needleman (1995, 1997), and Enakoutsa (2007, 2007) where the classical Gurson (1977)'s model was modified by replacing the (local) damage variable it involves with a nonlocal variable which includes a weight function, the so-called integral-nonlocal method. This type of nonlocal modeling technique has shown its potential to eliminate the pathological mesh size effects encountered in the finite element solution of structure problems involving ductile fracture. However, due to its heuristic nature, the integral nonlocal method is of less satisfaction, especially from the theoretical point of view. This has motivated the development of a more elaborated nonlocal model based on micromechanical foundations, the Gologanu-Leblond-Perrin-Devaux (GLPD) micromorphic model, Gologanu, Leblond, Perrin, and Devaux (1997).

The GLPD model was derived from an extension of Gurson (1977)'s original homogenization procedure, based on conditions of homogeneous boundary strain rate, to conditions of *inhomogeneous* boundaries strain rate. In Gologanu et al. (1997) model, the velocity imposed on the boundary of the representative cell considered is no longer linear but quadratic with respect to the coordinates. Doing so allows to account for sharp gradient of the macroscopic strain rate encountered, for instance, near crack tips or during strain localization. The output of the procedure was a model of "micromorphic" type, involving the second gradient of the macroscopic velocity and a generalized macroscopic stress of "moment" type along with some microstructural characteristic distance of the order of the average voids semi-spacing. To grasp more and more physical understandings of this model requires to apply it to simple boundary value problems involving various loading paths and boundary conditions. So far, the GLPD model was used to solve problems including the circular bending of beam in plane strain, hollow sphere and cylinder subjected to axisymmetric loading conditions, see Enakoutsa (2007, **?**) and Enakoutsa (2012, 2013, 2013). These analytical tools were useful to assess the reliability of the numerical implementation of the GLPD model into SYSTUS, a finite element code developed by Engineering Systems International Group. They have also been used to illustrate the physical soundness of the GLPD model. The objective of this paper is to follow up the study of the applications of the GLPD model to simple boundary value problems. The problem model[1] considered here is a thin-walled tube subjected to combined tension and torsion loading conditions; this tube is supposed to be made of a plastic metal described by the GLPD model. A similar problem was considered in the work of Collin, Caillerie, and Chambon (2009) and Gao (2004), but the loading conditions and the material constitutive laws were different.

The paper is organized as follows.

- Section 2 provides a summary of the constitutive equations of the GLPD model. A short review of the micromechanical foundations of the model is also presented.
- Section 3 solves the boundary value problem considered using the (classical) von Mises plasticity model, as a first step. Explicit expressions for the components of the stress tensor are obtained through a special parametrization of the von Mises yield criterion.
- Next, Section 4 extends the procedure of solution developed in Section 3 to the case where the matrix material of the tube is made of GLPD micromorphic model. Here also explicit expressions for the ordinary and higher-order stress as well as the formulae for the strain rate and its gradient are provided. It is found that these expressions depend on the characteristic length scale the GLPD micromorphic model involves.
- Finally, Section 5 discusses the newly derived solution.

2. A review of the GLPD model

The derivation of the GLPD model is based on the homogenization procedure of a representative volume element (RVE) subjected to inhomogeneous boundary strain rate. The outcome of the procedure involves an ordinary macroscopic Cauchy stress $\Sigma(\mathbf{X})$ and a macroscopic third-rank generalized stress $\mathbf{M}(\mathbf{X})$ of moment type defined as

$$\begin{cases} \Sigma_{ij}(\mathbf{X}) = <\sigma_{\mathbf{ij}}(\mathbf{x})>_{\mathcal{C}(\mathbf{X})} \\ \mathbf{M_{ijk}}(\mathbf{X}) = <\sigma_{\mathbf{ij}}(\mathbf{x})(\mathbf{x_k} - \mathbf{X_k})>_{\mathcal{C}(\mathbf{X})}, \end{cases} \tag{1}$$

[1] The problem model of thin-walled tube is of high interest to pipelines' design community where the knowledge of the stress state in the pipe is a key point.