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Analytical study on torsion of shape-memory-polymer prismatic bars with rectangular cross-sections

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ABSTRACT

In this paper, the response of shape memory polymer (SMP) bars with rectangular cross-sections under torsional loadings is analytically studied. To this end, we first reduce the recently proposed small-strain 3D phenomenological constitutive model for SMPs to the shear case. Then, an analytical solution for torsional response of SMP rectangular bars in a full cycle of *stress-free strain recovery* is derived. We also implement the 3D constitutive equations in a finite element program and simulate a full cycle of *stress-free strain recovery* of a rectangular SMP bar. Analytical and numerical results are then compared showing that the analytical solution gives, besides the global load–deflection response, accurate stress distributions in the cross-section of the rectangular SMP bar. Some case studies are also presented to show the validity of the presented analytical method. Results are compared with the experimental data recently reported in the literature which showing an agreement between the predicted results and experiments. The analytical solution can also be used for analysis of helical springs in which both the curvature and pitch effects are negligible. This is the case for helical springs with large ratios of mean coil radius to the cross-sectional equivalent radius (spring index) and also small pitch angles. Using this solution simplifies the analysis of the helical springs to that of the torsion of a straight bar with rectangular cross-section.

1. Introduction

Since the first observation of the shape memory effect in some polymers, research on SMPs has been an active field. Recently, manufacturing of SMP devices, has considerably increased, thanks to their unique ability in recovering large stored strains (Lendlein et al., 2009; Lendlein & Langer, 2002; Liang, Rogers, & Malafeew, 1997; Barot, Rao, & Rajagopal, 2008).

SMPs have been researched, developed, and utilized in a wide range of applications such as advanced technologies in the aerospace, medical, microelectromechanical systems (MEMS) and oil exploration industries (Díaz Lantada et al., 2010; Monkman, 2000; Ghosh, Reddy, & Srinivasa, 2012). Compared to other smart materials such as shape memory alloys, SMPs have ability of large elastic deformation, low energy consumption for shape programming, potential biocompatibility, low cost, low density, biodegradability and excellent manufacturability (Baghani, Naghdabadi, & Arghavani, 2012b, 2012a; Beloshenko, Varyukhin, & Voznyak, 2005; Cheng & Li, 2008; Jarali, Raja, & Upadhya, 2010; Sar, 2010; Ghosh & Srinivasa, 2011).

Despite of all advantages mentioned in the above, the much lower stiffness of un-reinforced SMPs prevents them from practical applications in cases where a large recovery stress is required (e.g., as actuators). To overcome such disadvantage, different reinforced-SMP composites have been developed and utilized (Li & Wang, 2011; Xu & Li, 2010).

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Up to now, the characterization of SMP behavior has been carried out with traditional uniaxial tension and compression tests (Liu, Gall, Dunn, Greenberg, & Diani, 2006; Xu & Li, 2010), however torsional thermomechanical tests can play an important role in the characterization of an SMP constitutive model. Besides, torsional test may be useful to verify and evaluate the validity of a 3D constitutive model as well as its numerical counterpart.

Recently, Diani et al. (2011) have performed a series of experiments on unconstrained recovery of rectangular bars made of SMPs. They also studied the effect of rate of applied temperature on the responses of SMPs. The necessity of obtaining an accurate analytical and numerical solution for the SMP devices, besides their new emerging applications, motivated the authors to seek analytical and numerical solutions for SMP rectangular bars. Although, analytical and numerical modeling of torsion of rectangular bars for some materials are presented in the literature, but to the knowledge of the authors, there are no relevant publications in this context for SMPs.

In this paper, employing the constitutive model recently presented by Baghani, Naghdabadi, Arghavani, and Sohrabpour (2012c), we present an analytical solution for SMP rectangular bars in a *stress-free strain recovery* process. In addition, to have a numerical tool for comparing the results of analytical solution with, we also develop a 3D finite element solution. Employing the finite element solution, we will verify the validity of assumptions used in the analytical solution as well as showing its accuracy. The solution is also validated by comparing the predicted results with experimental data reported in the literature. Moreover, it is observed that the solution time in the analytical method is much less than the computational time for the finite element simulations.

This solution can also be used for helical springs (with rectangular cross-sections) under axial loadings in which both the curvature and pitch effects are negligible. This is the case for helical springs with large ratios of mean coil radius to the cross-sectional equivalent radius (spring index) and also small pitch angles (such a spring is depicted in Fig. 1). Using this solution simplifies the analysis of the helical springs to that of the torsion of a straight bar with a rectangular cross-section.

This paper is organized as follows. In Section 2, a 3D constitutive equation for SMPs is briefly reviewed. In Section 3, the 3D constitutive relations are reduced to a constitutive equation for the cases in which only the shear strains and stresses exist. In the case of SMP rectangular bars, we also make use of the constitutive equations reduced to shear case and solve the torsion of rectangular bars analytically in a *stress-free strain recovery* cycle. In Section 4, finite element simulations are given and some case studies are reported for rectangular bars. In addition, a comparison is made between the proposed analytical solution, finite element simulations and experimental data available in the literature. Finally, we present a summary and draw conclusions in Section 5.

2. A 3D constitutive model for SMPs

In this section, a typical cycle of an SMP under thermomechanical loadings is described. A 3D constitutive model for SMPs is then briefly reviewed.

From a macroscopic point of view, shape memory effect can be characterized in a stress–strain-temperature diagram as illustrated in Fig. 2. The thermomechanical cycle starts at a strain- and stress-free state while temperature is high, T_h (point ⓐ, permanent shape). At this point, a purely mechanical loading is applied to SMP and the material demonstrates a rubbery behavior up to point ⓑ. At point ⓑ, strain is held fixed (external loadings) and the temperature is decreased until the rubber-like polymer drastically turns into a glassy polymer at a low temperature T_l (point ⓒ, fixed shape). In fact, in the neighborhood of the transition temperature T_g, SMP exhibits a combination of rubbery and glassy behaviors. Subsequently, the material is unloaded. Regarding the much higher stiffness of the glassy phase in comparison to the rubbery phase, after unloading, strains change slightly (point ⓓ). Finally, we increase the temperature up to T_h. It is seen that the strain will relax and the original permanent shape can be recovered (point ⓐ). This cycle is called a *stress-free strain recovery* in SMP applications. In practice, other types of recoveries may happen. If at point ⓓ, the strain is fixed and the temperature is increased, the *fixed-strain stress recovery* (point ⓔ) happens. Dotted line in Fig. 2, illustrates the mentioned behavior.

We now use an equivalent representative volume element (RVE) of the material composed of a *glassy* phase, a *rubbery* phase and a hard segment (Fig. 3) to derive a 3D SMP constitutive equation. It is assumed that the volume fraction of the

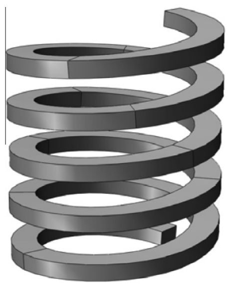

Fig. 1. A helical spring with rectangular cross-section.