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Microfluid-induced nonlinear free vibration of microtubes

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ABSTRACT

The microfluid-induced nonlinear free vibration of microtubes is studied in this paper. A derivation of the nonlinear equation of motion is presented based on Hamilton's principle and a modified couple stress theory. The geometric nonlinearity, arising from the mid-plane stretching, is taken into account. The modified couple stress theory is used to capture the micro-structure dependent size effects when the microtubes are at micron- and submicron scales. The static postbuckling problem is then studied and the size-dependent post-buckling configurations are analyzed. The approximate solution to the nonlinear free vibration is obtained using the homotopy analysis method. The influences of internal material length scale parameter, outer diameter, flow velocity, and Poisson's ratio on the dynamic behavior are discussed in detail.

1. Introduction

Microtubes have many potential applications in semiconductor technology, information technology, and biology. For example, the hollow geometry can be considered to deliver drugs in targeted cancer therapy which results in a rapid decrease in tumour size (Bhirde et al., 2009). Flow-containing micropipes also can be utilized to be a class of microresonators (Najmzadeh, Haasl, & Enoksson, 2007).

Over the past decades, the linear and nonlinear dynamics of macropipe conveying fluid have been studied extensively using the classical continuum theory (Folley & Bajaj, 2005; Ghayesh & Païdoussis, 2010; Hellum, Mukherjee, & Hull, 2011; McDonald & Namachchivaya, 2005; Paidoussis, 1998; Paidoussis & Issid, 1974). In recent years, there has been a great deal of interest in static and dynamic behavior of pipe at micro and nano scales (Rinaldi, Prabhakar, Vengallatore, & Païdoussis, 2010; Wang, 2010; Wang, Ni, Li, & Qian, 2008; Xia & Wang, 2010; Yan, He, Zhang, & Wang, 2009; Yoon, Ru, & Mioduchowski, 2005). However, the classical theory cannot be directly used to interpret the size-dependent behaviors which were often observed in experiments (Fleck, Muller, Ashby, & Hutchinson, 1994; Lam, Yang, Chong, Wang, & Tong, 2003). Thus, several non-classical elasticity theories have been developed to incorporate the size dependence. The modified couple stress theory, presented by Yang, Chong, Lam, and Tong (2002), is one of the non-classical continuum theories which contains only one additional material parameter that can capture the size effect. The modified theory has been applied to study the static and dynamical behavior of beam, rod, and plate at micro scale. Park and Gao (2006) utilized the theory to analyze the static mechanical properties of an Euler–Bernoulli beam, their analytical results successfully explained the outcomes of epoxy polymeric beam bending experiments. Ma, Gao, and Reddy (2008) developed a microstructure-dependent Timoshenko beam model. Based on their linear model, Asghari, Kahrobaiyan, and Ahmadian (2010) presented a nonlinear Timoshenko beam formulation base on the modified couple stress theory. In their work, stretching of mid-plane was incorporated in the mathematical formulation to take into account the influence of geometric non-linearity. Tsiatas

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(2009) and Jomehzadeh, Noori, and Saidi (2011), respectively studied the microscale vibration of microplates. Other models including functionally graded microbeam (Ke, Wang, Yang, & Kitipornchai, 2011), composite laminated beam (Chen, Chen, & Sze, 2012) were developed based on the same theory.

Recently, intense research activities have been focused on the microfluid-induced linear and nonlinear vibration of the microtubes. For example, Rinaldi et al. (2010) modeled a class of microresonators that may be characterized as micropipes containing internal fluid, they derived the linear equation of motion. Wang (2010) presented a new linear model for the vibration analysis of fluid-conveying microtubes based on the modified couple stress theory. In that paper, the micro-structure dependent size effect was taken into account. Xia and Wang (2010) formulated the microfluid-induced vibration and instability using the non-classical Timoshenko theory. Kahrobaiyan, Asghari, Hoore, and Ahmadian (2011) investigated the size-dependent nonlinear forced vibration of an Euler–Bernoulli microbeam. Their results shown that the nonlinearity can significantly change the static and vibration behavior of the microbeam. Fu and Zhang (2010) investigated the bending and the buckling behavior of a hollow microtubule. They observed that the size-dependent mechanical behavior is quite different from those obtained from the non-local elasticity theories. However, to the best of our knowledge, no nonlinear vibration model of microtubes is available to date. Based on above mentioned pioneering work, here we present a nonlinear model to account for mid-plane stretching of the tube at micro scale.

2. Equations of motion

We start with a brief introduction to the modified couple stress theory, in which the strain energy Q in a deformed isotropic linear elastic material occupying region Ω can be written as

$$Q = \frac{1}{2}\int_{\Omega}\left(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij}\right)d\upsilon \quad (i,j = 1,2,3), \tag{1}$$

where σ_{ij} is the Cauchy stress tensor, ε_{ij} is the strain tensor, χ_{ij} is the symmetric curvature tensor, m_{ij} is the deviatoric part of the couple stress tensor, and $d\upsilon$ is a volume element. The tensors satisfy the geometrical equations:

$$\varepsilon_{ij} = \frac{1}{2}\left[\nabla u_i + (\nabla u_i)^T\right], \quad \chi_{ij} = \frac{1}{2}\left[\nabla\theta_i + (\nabla\theta_i)^T\right], \tag{2}$$

where u_i is the displacement vector, θ_i is the rotation vector defined by

$$\theta_i = \frac{1}{2}\mathrm{curl}(u_i). \tag{3}$$

The constitutive equations are

$$\sigma_{ij} = \lambda \mathrm{tr}(\varepsilon_{ij})\delta_{ij} + 2G\varepsilon_{ij}, \tag{4}$$

$$m_{ij} = 2l^2 G\chi_{ij}, \tag{5}$$

where λ and G are Lamé's constants (G is also known as the shear modulus), δ_{ij} is the Kronecker's delta function. l is the internal material length scale parameter, which can capture the size effect. According to the modified couple stress theory, it is clear that the size-dependent behavior is an inherent property of a microbeam when its characteristic size, i.e., the thickness or the diameter of a tube, is comparable to the internal material length scale l. Based on this theory, Ma et al. (2008) and Park and Gao (2006), respectively derived the governing equation of Bernoulli–Euler beam and Timoshenko beam at micro scale. In this paper, we focus on the dynamical behavior of the microtube conveying fluid.

The system under consideration is a straight and slender microtube with length L, flexural rigidity EI, and mass per unit length m, as shown in Fig. 1. The internal flow in the microtube is due to an incompressible fluid of mass per unit length M flowing with velocity V_f. The Bernoulli–Euler beam theory is adequate to describe the dynamics of the sufficiently slender microtube. Thus, the displacement field can be assumed to be

$$U = -Z\psi(X,T), \quad V = 0, \quad W = W(X,T), \tag{6}$$

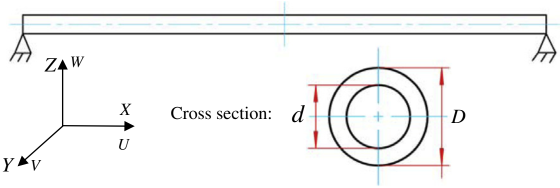

Fig. 1. Schematic of a fluid-conveying microtube.