International Journal of Engineering Science 76 (2014) 56–72

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

A numerical study of blood flow using mixture theory

Wei-Tao Wu [a], Nadine Aubry [b], Mehrdad Massoudi [c,*], Jeongho Kim [d], James F. Antaki [d]

[a] Department of Mechanical Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, USA
[b] Department of Mechanical Engineering, Northeastern University, Boston, MA 02115, USA
[c] U. S. Department of Energy, National Energy Technology Laboratory (NETL), P.O. Box 10940, Pittsburgh, PA 15236, USA
[d] Department of Biomedical Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, USA

ARTICLE INFO

Article history:
Received 17 September 2013
Received in revised form 10 December 2013
Accepted 13 December 2013
Available online 9 January 2014

Keywords:
Blood flow
Mixture theory
Two phase flow
Rheology
Channel flow
Non-linear fluids

ABSTRACT

In this paper, we consider the two dimensional flow of blood in a rectangular microfluidic channel. We use Mixture Theory to treat this problem as a two-component system: One component is the red blood cells (RBCs) modeled as a generalized Reiner–Rivlin type fluid, which considers the effects of volume fraction (hematocrit) and influence of shear rate upon viscosity. The other component, plasma, is assumed to behave as a linear viscous fluid. A CFD solver based on OpenFOAM® was developed and employed to simulate a specific problem, namely blood flow in a two dimensional micro-channel, is studied. Finally to better understand this two-component flow system and the effects of the different parameters, the equations are made dimensionless and a parametric study is performed.

Published by Elsevier Ltd.

1. Introduction

Blood is a unique multi-component fluid whose composition is responsible for important rheological properties that are responsible for its vital physiological functions. Its primary constituents are flexible, discoid red blood cells (RBCs) (approximately 45% volume fraction) suspended within essentially Newtonian plasma (Robertson, Sequeira, & Kameneva, 2008). In the context of blood-wetted medical devices, the trafficking of RBCs within the plasma greatly contributes to both safety and efficacy. For example in oxygenators and artificial lungs, it is desirable for RBCs which are responsible for transport of oxygen and carbon dioxide, to efficiently interact with the artificial fibers that deliver and remove these gasses. In virtually all blood-wetted devices, it is usually undesirable for the blood to coagulate on the artificial surfaces. This phenomenon, known as thrombosis, is mediated by the platelets, a dilute constituent of blood which is in turn influenced by collisions with the RBCs. Therefore at the microscopic level, the distribution of RBCs is responsible for the distribution of platelets (Aarts, Steendijk, Sixma, & Heethaar (1986)). Accordingly the design of improved cardiovascular devices requires an accurate model of these phenomena. Conversely, the inadequacies of current models stifle our ability to design these devices with any confidence (Thompson, Loebe, & Noon, 2003). As a result, contemporary designs are based primarily on empiricism, and experimental trial-and-error.

It is known that in large vessels (whole) blood behaves as a Navier–Stokes (Newtonian) fluid (Fåhraeus, 1929; Fåhræus & Lindqvist, 1931; Fung, 1993, Chapter 3); however, in a vessel whose characteristic dimension is in the range of tens to hundreds of blood cells (e.g., for a diameter in the range of 20–500 microns) blood behaves as a non-Newtonian fluid, exhibiting shear-thinning, stress relaxation (Bagchi, 2007) and phase separation (Goldsmith, 1971). In larger vessels blood also exhibits shear thinning, particularly for shear rates below 100 s^{-1}. These non-Newtonian properties of blood are mainly attributed to

* Corresponding author.
E-mail address: Mehrdad.Massoudi@NETL.DOE.GOV (M. Massoudi).

0020-7225/$ - see front matter Published by Elsevier Ltd.
http://dx.doi.org/10.1016/j.ijengsci.2013.12.001

the behavior of the RBCs: the aggregation and disaggregation of the RBCs as a function of shear rate; the deformability of the RBCs; and the alignment of the RBCs in response to extensional flow. Because of the fibrinogen and large globulins, RBCs aggregate and form rod-shaped stacks called rouleaux at low shear rate [see Popel and Johnson (2005) and Bäumler, Neu, Donath, and Kiesewetter (1999)]. This aggregation of the RBCs, which increases the blood viscosity is however reversed when the shear rate increases, causing their disaggregation. The volume fraction of RBCs (known as *hematocrit*) strongly influences all of the aforementioned phenomena. Increased packing of RBCs affects their collision frequency, and hence their ability to aggregate and to migrate within the flow field. Accordingly, the viscosity of the blood increases dramatically as the hematocrit increases (Chien, Usami, Taylor, Lundberg, & Gregersen, 1966, 1971; Pries, Neuhaus, & Gaehtgens, 1992). Likewise, the property of shear-thinning viscosity becomes weaker and eventually disappears as the hematocrit decreases (Brooks, Goodwin, & Seaman, 1970). The deformability of RBCs is also an important property which affects viscosity and cell trafficking. In capillaries, with sizes equivalent or smaller than that of RBCs, the deformability of RBCs allow them to fold and flex as they transport gasses through the vessel walls. In larger vessels or passages, the deformability of the RBCs allow them to become more streamlined, and aligned at high shear rates – thereby contributing to shear thinning (Chien, 1970). It should be acknowledged that, although the RBCs dominates the rheological properties of blood, other factors such as the plasma viscosity, white blood cells, etc. also play a role (Kameneva, Garrett, Watach, & Borovetz, 1998; Middleman, 1972; Rourke & Ernstene, 1930).

In the past several decades, investigations of blood flow in micro-scale channels have revealed several important phenomena due to the complex rheological properties of blood. In vessel of diameter ranging from approximately 0.05 to 1.5 mm, blood exhibits a thin layer adjacent to the wall that is depleted of RBCs (Marhefka et al., 2009). This phenomenon is known as the *Fahraeus–Lindqvist effect* (Fung, 1993). This depletion of RBCs near the wall causes the hematocrit of branch vessels to be depleted – a phenomenon known as *plasma-skimming* (Carr & Wickham, 1990; Krogh, 1921; Marhefka et al., 2009; Skalak, Ozkaya, & Skalak, 1989). Several early experiments related to blood cell margination were performed in tubes, such as Goldsmith's seminal experiment in which he flash froze the flow to observe the concentration of cells to the centerline (Goldsmith, 1968), a great number of modern experiments are performed in microchannels of rectangular cross section. The reason for this is twofold: microchannels can be much more easily formed using photolithography, and secondly, parallel walls are much more amenable to microscopic measurements. The technology for visualizing flow in non-parallel (e.g. circular) cross sections is still in its infancy, and include micro PIV (Sugii & Okuda, 2005) and confocal PIV methods (Lima & Wada, 2006; Patrick, Chen, Frakes, Dur, & Pekkan, 2011). Furthermore, micro-channels have also been used in a variety of devices such as film oxygenators and recently, dialysis-like cell separators such as the one being developed by our group for treatment of malaria-infected blood. [see Kim, Massoudi, Antaki, and Gandini (2012)]. In summary, it is evident that blood flow at micro-scale exhibits more complex behavior and acts as a multi-component material, which cannot be described by a single phase model.

Motivated by the observation of these phenomena, various multiphase models for blood have been developed. The Immersed Boundary Method (IBM) combined with the Lattice Boltzman Method (LBM) has become a popular method for modeling deformation, cluster formation and collisions of RBCs [see Dupin, Halliday, Care, Alboul, and Munn (2007), Clausen, Reasor, and Aidun (2010) and Zhang, Johnson, and Popel (2009)]. An alternative method consists of the so-called two-fluid or Eulerian–Eulerian two phase model (Jung, Hassanein, & Lyczkowski, 2006). In tandem, over the past four decades, multi-constituent models have been developed for a variety of non-biologic fluids. Two methods in particular are based on first principles of continuum mechanics: the Mixture Theory (or the theory of interacting continua) [see Rajagopal and Tao (1995)] and the Averaging Method [see Ishii (1975)]. In this paper the Mixture Theory is applied as a basis for deriving a two-phase model for blood (Massoudi, Kim, & Antaki, 2012).

The Mixture Theory was first presented by Truesdell in 1957 (Truesdell, 1957) as a means of generalizing the equations and principles of the mechanics of a single continuum to include any number of superimposed continua. In a sense, it is a homogenization process in which each component is regarded as a single continuum and at each instant of time, every point in space is considered to be occupied by a particle belonging to each component of the mixture (Truesdell, 1984). In recent years it has been applied to a variety of applications such as fluid–solid particles, lubrication with binary-mixtures of bubbly oil, viscoelastic porous mixtures, swelling porous media with microstructure, reacting immiscible mixtures, polymeric solutions, growth and remodeling of soft tissues, ionized fluid mixtures, etc., (Massoudi, 2008). For review articles on this subject, see the papers by Atkin and Craine (1976a, 1976b) and Bowen (1976). Mixture Theory has also been used in a variety of biomechanics applications [see for example, Ateshian, Likhitpanichkul, and Hung (2006), Garikipati, Arruda, Grosh, Narayanan, and Calve (2004), Humphrey and Rajagopal (2002), Klisch and Lotz (2000), Lemon, King, Byrne, Jensen, and Shakesheff (2006) and Tao, Humphrey, and Rajagopal (2001)]. Because of its many desirable properties, Mixture Theory has been adopted for this study.

The paper is organized as follows. The kinematical variables and governing equations are introduced in Section 2 while the related constitutive equations are presented in Section 3. In Section 4, the effect of the various dimensionless parameters of our model is studied.

2. Governing equations

Let $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ denote the position of the bodies in the reference configuration (i.e., prior to mixing) belonging respectively to the plasma, treated as a fluid constituent, and the RBCs, treated as a solid. The motion of the two components can be represented as [see Johnson, Massoudi, and Rajagopal (1991)]