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Small-scale indentation of an elastic coated half-space: Influence of Poisson's ratios on the substrate effect

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ABSTRACT

Frictionless indentation of an elastic layer attached to an elastic half-space is considered in the small-scale contact range. Based on the first-order asymptotic solution, a simple analytical approximation is suggested for the indentation scaling factor that takes into account the elastic layer's finite thickness as well as the effect of the elastic substrate. The influence of both Poisson's ratios on the substrate effect is studied in detail, and the error of the 1/10 rule of Buckle is estimated for a relatively large range of the elastic moduli mismatch.

1. Introduction

A coating material deposited onto a substrate of another material represents one of the most important material configurations (Chen & Vlassak, 2001). In recent years, there has being an increasing interest to accurate and robust measurements of the elastic properties of thin films by means of nanoindentation techniques (Antunes, Fernandes, Sakharova, Oliveira, & Menezes, 2007; Hemmouche et al., 2013; Huang & Chang, 2010). Because thin films are usually deposited on elastic substrates, a non-trivial problem arises to extract intrinsic mechanical properties of the film from the so-called equivalent (effective or composite) elastic modulus, E_{eq}, provided by the Oliver–Pharr method originally developed for isotropic homogeneous semi-infinite elastic solids (Fischer-Cripps, 2004).

Based on phenomenological arguments, a number of empirical models have been proposed to relate the equivalent modulus E_{eq} to the elastic moduli of the film, E_f, and that of the substrate, E_s, in the following reciprocal forms:

$$E_{\mathrm{eq}} = E_s + (E_f - E_s)\Phi \tag{1}$$

or

$$E_{\mathrm{eq}} = E_f + (E_s - E_f)\Psi \tag{2}$$

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Here, Φ and $\Psi = 1 - \Phi$ are weight functions which depend on the ratio of the contact radius, a, or the indentation depth, h, to the film thickness, t. At that, it is assumed that $\Phi \to 0$ when $a/t \to \infty$ and $\Phi \to 1$ when $a/t \to 0$ (the substrate effect is eliminated).

To estimate the weight functions Φ and Ψ, a number of analytical approaches have been proposed. The model developed by Gao, Chiu, and Lee (1992) is based on a first-order perturbative solution under the assumption that the mechanical properties of the film and substrate do not differ greatly. Namely, Eq. (1) like formulas were suggested in Gao et al. (1992) for the effective Poisson's ratio, $\nu_{\rm eff}$, and the effective shear modulus, $G_{\rm eff}$. It was shown (Chen & Vlassak, 2001) that the model agrees with the results of finite element simulations for the moduli mismatch ratio $\eta = E_s/E_f$ in the range 0.5–2. Based on the analytical solution (Li & Chou, 1997) for the Green function for an elastic isotropic coated half-space, a relatively simple approximation for the function Ψ was developed by Perriot and Barthel (2004) in a wider range of the moduli mismatch ($\eta = 0.01 - 100$).

In the present work, we pursue an alternative approach, which originates from the methodology introduced by Hayes, Keer, Herrmann, and Mockros (1972) for accounting for the effect of absolutely rigid substrate in the frictionless flat-ended cylindrical indentation. Note that the analytical solution obtained in Hayes et al. (1972) was based on the integral transform technique proposed by Lebedev and Ufliand (1958) for solving the axisymmetric contact problem for an elastic layer. In the case of a cylindrical indenter, the contact radius remains constant during indentation and the indentation stiffness is represented by the formula

$$\frac{P}{h} = 2aE_f^*\kappa \tag{3}$$

Here, P and h are the contact force and the indenter displacement, respectively, E_f^* is the reduced elastic modulus of the elastic layer given by

$$E_f^* = \frac{E_f}{1 - \nu_f^2} \tag{4}$$

Eq. (3) introduces the so-called indentation scaling factor $\kappa(a/t, \nu_f)$, which takes into account the thickness effect while depending on the ratio a/t of the contact radius to the layer thickness and Poisson's ratio of the elastic layer, ν_f.

In the case of an elastic film deposited onto an elastic substrate, Eq. (3) can be generalized straightforwardly as follows (Argatov, 2010):

$$\frac{P}{h} = 2aE_f^*k \tag{5}$$

Here, $k(a/t, \nu_f, \nu_s, \eta)$ is the corresponding indentation scaling factor, which accounts for the effect of the substrate, $\eta = E_s/E_f$ is the elastic moduli mismatch ratio. Eq. (5) assumes (1) linear elastic deformation response of the coated system, (2) frictionless contact between the indenter and the surface of the elastic film, (3) constant contact area during indentation.

The effect of Poisson's ratio on the indentation scaling factor κ for an isotropic elastic layer attached to a rigid substrate was recently investigated in Argatov, Guinovart-Díaz, and Sabina (2012). It was shown that the case of negative Poisson's ratio is very important for both the quasi-static and dynamic indentation compliances. Recall that materials with a negative Poisson's ratio are known as auxetic materials (Evans & Alderson, 2000; Yang, Li, Shi, Xie, & Yang, 2004). Indentation compliance for a layered elastic medium composed of alternating layers of negative and positive Poisson's ratio was considered in Kocer, McKenzie, and Bilek (2009).

Below, we study the effect of both Poisson's ratios on the indentation scaling factor k in the small-scale contact range. It should be noted that though the case of extremely small contacts has its own importance (Liu, Gu, & Huang, 2011), the results obtained below can be used when the contact radius is about a half of the film thickness.

2. Indentation scaling factor, equivalent elastic modulus, and weight functions

Recently, based on the generalization of the theorem for the incremental indentation stiffness proved by Barber (2003) in the case of frictionless indentation of an elastic half-space, it was shown (Argatov & Sabina, 2013) that for an arbitrary axisymmetric blunt indenter (when the contact area is a circle of radius a), the incremental indentation stiffness can be evaluated as

$$\frac{dP}{dh} = 2aE_f^*k \tag{6}$$

It is to emphasize that the same indentation scaling factor enters both Eqs. (5) and (6).

Thus, since the application of the Oliver–Pharr method allows to evaluate experimentally the left-hand side of Eq. (6), the problem of the material parameter identification now relies on the analysis of the indentation scaling factor $k(\varepsilon, \nu_f, \nu_s, \eta)$.

By definition of the equivalent reduced elastic modulus, formula (6) can be rewritten in the form

$$\frac{dP}{dh} = 2aE_{\rm eq}^* \tag{7}$$