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Axisymmetric postbuckling analysis of size-dependent functionally graded annular microplates using the physical neutral plane

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ABSTRACT

This paper investigates the axisymmetric postbuckling of functionally graded material (FGM) annular microplates based on the modified couple stress theory, Mindlin plate theory and von Kármán geometric nonlinearity. Material properties are assumed to be graded in the thickness direction according to Mori–Tanaka homogenization method. By using the physical neutral plane, the bending-extension coupling is eliminated in both nonlinear governing equations and boundary conditions of FGM microplates. The differential quadrature (DQ) method is employed to discretise the governing equations, which are then solved to obtain the postbuckling path of FGM microplates with different boundary conditions. Numerical results are presented to highlight the effects of length scale parameter, gradient index, inner-to-outer radius ratio and radius-to-thickness ratio on the postbuckling characteristics of FGM microplates. It is found that FGM microplates do exhibit the bifurcation-type buckling when the applied in-plane compressive load acts along the physical neutral plane.

1. Introduction

Functionally graded materials (FGMs) are inhomogeneous composites whose material properties vary gradually with respect to spatial coordinates (Suresh & Mortensen, 1998). The material composition can be designed so as to improve the strength, toughness, high temperature withstanding ability, etc., to meet the desired structural performance in different engineering applications. With the development of advanced materials science and technology, FGMs have been employed in MEMS/NEMS and AFMs to achieve high sensitivity and desired performance (Fu, Du, Huang, Zhang, & Hu, 2004; Hasanyan, Batra, & Harutyunyan, 2008; Witvrouw & Mehta, 2005). Microbeams and microplates are the fundamental elements widely used in MEMS, NEMS and AFM applications with the order of microns or sub-microns. At this scale, the structures exhibit size-dependent deformation behavior which has been observed by many investigators in experiments, for example, the micro-torsion test of thin copper wires (Fleck, Muller, Ashby, & Hutchinson, 1994), the micro-bending test of thin nickel beams (Stolken & Evans, 1998), and the micro-bending test of epoxy polymeric beams (Lam, Yang, Chong, Wang, & Tong,

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2003). It is therefore of prime importance that the size effect resulting from the underling microstructures is taken into consideration in both theoretical and experimental studies of microscale FGM structures.

Due to the lack of an internal material length scale parameter, classical continuum theories cannot interpret and predict the size dependent effect in structures at micron- and sub-micron-scale. Therefore, utilization of higher order theories, such as the modified couple stress theory (Yang, Chong, Lam, & Tong, 2002), strain gradient theory (Lam et al., 2003), micropolar theory (Eringen, 1967), nonlocal elasticity theory (Eringen, 1972), and so on, which contain internal material length scale parameters, is necessary. For the use of these higher-order theories, the key is to identify of the length scale parameter in the micro- or nano-scale structures. We will make a brief review of the existing knowledge on such parameter in these higher-order theories.

In the nonlocal elasticity theory, Eringen (1983) proposed the nonlocal parameter $e_0 = 0.39$ by matching the dispersion curves via nonlocal theory for plane wave and Born–Karman model of lattice dynamics applied at the Brillouin zone boundary. Zhang, Liu, and Wang (2004) estimated the nonlocal parameter $e_0 = 0.82$ by matching the buckling strain obtained by the nonlocal thin shell model of CNT to those obtained by the molecular mechanics simulation. So far, no experiments have been conducted to determine the value of e_0. More in detail, Narendar and Gopalakrishnan (2011) tabulated the values of the nonlocal parameter obtained by various investigators using different methods. In the strain gradient theory (Lam et al., 2003), there are three length scale parameters l_0, l_1 and l_2 associated with the dilatation gradient, deviatoric gradient and rotation gradient, respectively. Lam et al. (2003) conducted the bending test of the homogeneous epoxy beam by using a nano-indenter to determine length scale parameters. These parameters can be also determined from torsion tests of slender cylinders (Chong, Yang, Lam, & Tong, 2001) in the micron scale. However, the modified couple stress theory involved only one length scale parameter associated with the rotation gradient, which makes this theory easier to use. The length scale parameter can be obtained by setting $l_0 = l_1 = 0$ and $l_2 = l$ in Lam et al. (2003). From the bending test of the homogeneous epoxy beam, it can be taken as $l = 17.6\ \mu$m. In the micropolar theory, the length scale parameter is defined as $l = \sqrt{2\beta/E}$ in Huang, Yan, Yan, and Yang (2000) and $l = \sqrt{\beta/2G}$ in Ramezani, Naghdabadi, and Sohrabpour (2008) where β,E and G are the micropolar constant, Young's modulus and shear modulus, respectively.

Among these higher order theories, the modified couple stress theory proposed by Yang et al. (2002) has been widely accepted and employed to study the size-dependent mechanical behavior of micro-scale structures. These include the elastic bending (Tsiatas, 2009; Park & Gao, 2006; Chen, Li, & Xu, 2011; Ma, Gao, & Reddy, 2011), buckling (Fu & Zhang, 2010; Akgöz & Civalek, 2011; Mohammad-Abadi & Daneshmehr, 2014a, 2014b), linear free vibration (Jomehzadeh, Noori, & Saidi, 2011; Ma, Gao, & Reddy, 2008; Zhou, Wang, & Qin, 2012), nonlinear vibration (Asghari, 2012; Asghari, Kahrobaiyan, & Ahmadian, 2010a; Farokhi, Ghayesh, & Amabili, 2013;Wang, Lin, & Liu, 2013) and postbuckling (Xia, Wang, & Yin, 2010) problems of the size-dependent microbeams, microplates and microshells. It should be pointed out that the afore-mentioned studies are for microbeams, microplates and microshells made of homogeneous materials only.

Recently, the modified couple stress theory has been further extended to FGM structures and is employed to establish the size-dependent FGM microbeam and microplate models. Asghari, Ahmadian, Kahrobaiyan, and Rahaeifard (2010b), Reddy (Ke, Wang, Yang, & Kitipornchai, 2011Nateghi, Salamat-talab, Rezapour, & Daneshian (2012a), Salamat-talab, Nateghi, & Torabi (2012), Salamat-talab et al. (2012), Şimşek, Kocatürk, & Akbaş (2013), and Şimşek et al. (2013) investigated the bending, free vibration, buckling, postbuckling, dynamic stability, and nonlinear vibration of FGM micro-beams based on Euler–Bernoulli, Timoshenko and third-order shear deformation beam theories. More importantly, Reddy & Kim (2012) made an important contribution by developing a general nonlinear third-order plate theory that accounted for geometric nonlinearity, size-dependent effect, and gradient in material properties. Reddy & Berry (2012) presented a size-dependent nonlinear theory for the axisymmetric bending of circular FGM microplates with power-law variation along the thickness direction. Thai and his co-workers discussed the size-dependent bending, buckling and free vibration of FGM microplates by using Kirchhoff plate theory, Mindlin plate theory and Reddy's higher-order plate theories and based on the modified couple stress theory (Thai & Kim, 2013; Thai & Vo, 2013; Thai & Choi, 2013). Ke, Yang, Kitipornchai, & Bradford (2012b, 2013) investigated the linear bending, buckling, free vibration and nonlinear vibration of FGM annular microplates based on the modified couple stress theory and Mindlin plate theory. Most recently, Komijani, Reddy, & Eslami (2014) analyzed the nonlinear thermo-electro–mechanical response of functionally graded piezoelectric micro-actuators.

These investigations mentioned above assumed that the undeformed plane is placed at the physical middle plane. Actually, the undeformed plane, i.e., the physical neutral plane, does not coincide with the geometric middle plane of FGM microbeams, microplates and shells due to the inhomogeneous material properties in the thickness direction. This has been brought to the attention of some investigators in FGM microscale and nanoscale structures. Asghari, Rahaeifard, Kahrobaiyan, & Ahmadian (2011) analyzed the bending and free vibration of FGM microbeams by using of the physical neutral plane. Eltaher, Alshorbagy, & Mahmoud (2013, 2014) investigated the bending, buckling and vibration of FGM nanobeams based on the physical neutral plane and nonlocal theory. Their studies showed that the use of the physical neutral plane can eliminate the bending-extension coupling in FGM structures.

In this paper, the postbuckling of FGM annular microplates is studied based on the modified couple stress theory, Mindlin plate theory and von Kármán geometric nonlinearity. It is assumed that the material properties are graded in the thickness direction and can be predicted according to Mori–Tanaka homogenization method. By using the physical neutral plane, the bending-extension coupling is eliminated in FGM microplates. A system of nonlinear algebraic equations is obtained by the principle of minimum total potential energy and is then solved by the differential quadrature (DQ) method and an iterative solution procedure to determine the postbuckling path (load–deflection curve) of FGM microplates with different boundary