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# Analysis of conical shells in the framework of coupled stresses theory

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In this paper, the thin conical shell model is developed by using the modified couple stress theory. This non-classical formulation can incorporate size effects in nano/micro scales. For this purpose, the thin shell model is used along with the modified couple stress theory, and the equations of motion with partial differentials and classical and non-classical boundary conditions are derived by using Hamilton's principle. Finally, the free vibrations of the single-walled carbon nanocone (SWCNC) are examined as a special case. The SWCNC is modeled as simply supported, and the Galerkin method is used to solve the vibrational problem. Results of the new formulation are compared to the classical theory. The results show that nanoshell rigidity is greater in the modified couple stress theory than in the classical theory, which leads to an increase in the natural frequencies. Also, the effect of size parameter on the vibration frequency of SWCNC in different lengths and apex angles is examined.

## 1. Introduction

Micro/Nano-scale mechanical elements, such as micro/nano-shells (cylindrical shells, conical shells, etc.) are the main components of micro/nano-scale systems such as Micro/Nano Electro Mechanical Systems (M/NEMS). Also, many carbon nanotubes are in the shape of conical shells. Single-walled carbon nanocones (SWCNCs) are made up of one-atom-thick carbon sheets shaped into hallow cones. The carbon sheets give SWCNCs unique electric, chemical, and mechanical properties (Ge & Sattler, 1994; Krishnan et al., 1997). Extensive application of SWCNCs in nano-devices, NEMS, sensors, and composites has brought these devices into the focus of researchers' attention (Shenderova, Lawson, Areshkin, & Brenner, 2001; Yeh, Chen, Hwang, Gan, & Kou, 2006). In recent years, researchers have used molecular dynamics (MD) simulation to study the mechanical properties of single-walled carbon nanocone (SWCNC). Using MD simulation, Wei et al. determined Young's modulus of the SWCNC in various diameters, heights, and apex angles (Wei, Liew, & He, 2007). In another study, Kumar et al. determined Young's modulus and shear modulus of the SWCNC in various diameters and heights as well as six different apex angles by using MD simulation. They estimated Young's modulus between 0.24 TPa and 0.73 TPa, and the shear modulus between 0.10 TPa and 0.29 TPa (Kumar, Verma, Bhatti, & Dharamvir, 2011). Hu et al. investigated the vibration of SWCNC using the classical theory. They modeled the SWCNC by using Euler–Bernoulli beam and Timoshenko beam models and compared their results with those of the MD simulation (Hu, Liew, He, Li, & Han, 2012). Lee et al. investigated the vibration of the

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single-walled carbon nanotube (SWCNT) and SWCNC by using the MD simulation and finite element method. They determined the mode shapes for various structures and supports of the SWCNC (Lee & Lee, 2012). Yan et al. examined the buckling of a SWCNC by using the MD simulation and determined the critical load for different radii and heights (Yan, Liew, & He, 2013).

Given the fact that MD simulation is costly and includes lengthy calculation, non-classical continuum theories such as the nonlocal theory, strain gradient theory and couple stress theory have recently been used to investigate mechanical properties and dynamic behaviors.

The classical couple stress theories, in which the higher order rotation gradients are incorporated as the deformation matrix, was presented by Toupin (1962), Mindlin and Tiersten (1962), Mindlin (1964) and Koiter (1964). Regarding the difficulties of determining length scale parameters, which classical couple stress theories consist of couple of them, Yang et al. proposed modified couple stress theories in which a new additional equilibrium equation and the equilibrium of the couples of moments, besides classical equilibrium equations of forces and their moments, exist and lead to one length scale parameter (Yang, Chong, Lam, & Tong, 2002). Afterwards, Mindlin introduced the general higher order stress theory by only considering the second order deformation gradient as supplementary deformation matrix, therefore the five linear elastic parameters are deduced from this additional part (Mindlin, 1965). Subsequently, Flack and Hatchinson modified Mindlin's formulation and named it as strain gradient theory (Fleck & Hutchinson, 1997). In this new formulation, the stretch gradient tensor and rotation gradient tensor, are taken into account as the constitutive parts of second order deformation tensor. Afterwards, the modified strain gradient theory, which considered only the symmetric parts of strain gradient in the equations, was proposed by Lam, Yang, Chong, Wang, and Tong (2003). According to this theory, the five length scale parameters presented by Mindlin was reduced to three length scale parameters. These three parameters can be combined and reduced to only one single measurable parameter under the assumptions of modified couple stress theory (Lam et al., 2003).

Gue et al. examined the longitudinal vibration of a SWCNC using the nonlocal theory. To this end, they used the beams theory to model the SWCNC and showed that an increase in the size parameter is accompanied by a decrease in natural frequencies (Guo & Yang, 2012). Firouz-Abadi et al. examined the vibration of SWCNC by using the thin shell model and the nonlocal theory. They showed that an increase in diameter, a decrease in apex angle, and a decrease in size parameter lead to an increase in the natural frequencies of SWCNC (Firouz-Abadi, Fotouhi, & Haddadpour, 2011). Ghorbanpour et al. investigated the nonlinear vibrations of the fluid-conveying SWCNC by using the nonlocal theory. For this purpose, they used Euler–Bernoulli beam model and showed that as the size parameter and fluid velocity increase, natural frequencies decrease (Ghorbanpour Arani, Kolahchi, Haghighi, & Barzoki, 2013b). Similarly, Fotouhi et al. studied the vibration of SWCNC with elastic foundation by using the nonlocal theory. In their experiment, they used the thin shell theory and showed that an increase in Winkler stiffness and Pasternak stiffness leads to a parallel increase in the natural frequencies of the SWCNC (Fotouhi, Firouz-Abadi, & Haddadpour, 2013).

Given the advancements made in nanoscience, there is a great necessity to investigate and model the conical shell which has the requisite features for the models used in the nanoscale. Therefore, in this study, the thin conical shell model is developed by using the modified couple stress theory. The new formulation has two main advantages for application in the nanoscale. These are being introduced for the first time.

The first advantage is that, in this formulation, the modified couple stress theory has been used and hence the developed formulation has the capability to model size effect in nano/micro scales. It is worth noting that the classical continuum theories lack the ability to incorporate size effect in micro/nano scales and, therefore, today, many researchers use higher order theories such as the modified couple stress theory and strain gradient theory to model components (Akgöz & Civalek, 2011; Asghari, Kahrobaiyan, & Ahmadian, 2010; Asghari, Kahrobaiyan, Rahaeifard, & Ahmadian, 2011; Ghorbanpour Arani, Bagheri, Kolahchi, & Khoddami Maraghi, 2013a; Kahrobaiyan, Asghari, & Ahmadian, 2013; Noghrehabadi et al., 2011; Şimşek & Reddy, 2013; Tadi Beni & Abadyan, 2013a, 2013b; Tadi Beni, Koochi, & Abadyan, 2011; Tadi Beni, Koochi, Kazemi, & Abadyan, 2012; Zeverdejani & Tadi Beni, 2013).

The second advantage of the new formulation is that it uses a more precise thin shell model. It goes without saying that in order to gain better results more precise models which are consistent with nanoscale components such as SWCNCs must be used. For this reason, the thin shell model was used to carry out more precise modeling of shell-shaped nanoscale components. This model is surely more appropriate than the beam model. Interestingly, researchers have recently used the shell model to model the SWCNT in a more appropriate way (Arash & Wang, 2012; Asghari, Rafati, & Naghdabadi, 2013; Fazelzadeh & Ghavanloo, 2012; Ghavanloo & Fazelzadeh, 2012; Ghorbanpour Arani, Amir, Shajari, & Mozdianfard, 2012; Ghorbanpour Arani, Zarei, Amir, & Khoddami Maraghi, 2013c; Jannesari, Emami, & Karimpour, 2012; Zeighampour & Tadi Beni, 2014a, 2014b).

It must be emphasized that, by employing higher order theories, the size effect can be modeled in nanoascale, but this modeling will be done by determining and using new length scale parameters in these theories. Today, many researchers have attempted to calculate the new parameters in higher order continuum theories. A simple practical and physical interpretation of the length scale parameter can be obtained in terms of elastic rigidity of cantilever nanobeam in the bending test. Based on the modified couple stress theory and considering the Euler beam model, the material length scale parameter can be directly related to the difference between the elastic modulus of the material (Park & Gao, 2006).

The material length scale parameters also might be determined via molecular dynamic simulation or experiments. Previous researchers used atomistic simulations and molecular dynamics to determine the size effect parameters (Chan & Zhao, 2011; Duan, Wang, & Zhang, 2007; Maranganti & Sharma, 2007). Maranganti and Sharma (2007) used an atomistic approach