International Journal of Engineering Science 81 (2014) 123–134

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

A novel generalized thermoelasticity model based on memory-dependent derivative

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ARTICLE INFO

Article history:
Received 22 January 2014
Received in revised form 18 April 2014
Accepted 22 April 2014
Available online 16 May 2014

Keywords:
Generalized thermoelasticity
Memory-dependent derivative
Fractional calculus
LS model
NILT

ABSTRACT

In this work, by introducing memory-dependent derivative (MDD), instead of fractional calculus, into the Lord and Shulman (LS) generalized thermoelasticity, we establish a new memory-dependent LS model, which might be superior to fractional ones: firstly, the new model is unique in the form, while the fractional order theories have different pictures within different authors; secondly, physical meaning of the former is more clear seeing the essence of MDD's definition; thirdly, the new model is depicted by integer order differential and integral, which is more convenient in numerical calculation compared to fractional ones; lastly, the *Kernel function* and *Time delay* of MDD can be arbitrarily chosen, thus, provides more approaches to describe material's practical response, as a consequence, it is more flexible in applications than fractional ones, in which the significant variable is the fractional order parameter. In numerical implementation, a one-dimensional semi-infinite medium with one end subjecting to a thermal loading is considered using the integral transform method. While in inverse transformation, an efficient and pragmatic algorithm 'NILT' is adopted. Parameter studies are performed to evaluate the effect of *Kernel function* and *Time delay* and a memory-dependent parameter is also defined. Finally, some concluding remarks are summarized.

1. Introduction

Compared to the coherent optical source of long pulses, subpicoseconds or femtoseconds ultrafast lasers have several prominent advantages, such as low input energy, high power output, limited spread of the heat affected zone and collateral damage, less debris contamination, and good reproducibility (Qi & Suh, 2010; Ren, Cheng, Chen, Zhang, & Tzou, 2013). As a consequence, interaction of ultrafast laser with matter has caught numerous theoretical and experimental research interest (Chen, Tzou, & Beraun, 2006) and it is the upsurge of investigation on ultrafast lasers that have excavated their numerous applications, such as: welding and drilling of metals, surface annealing, sintering of ceramics, micro-machining and fabrication of materials (Afrin, Zhang, & Chen, 2014; Gan & Chen, 2013; Hendijanifard & Willis, 2013; Qi & Suh, 2010). While rapid development of the novel laser burst technology has also been enlarging the application scope (Ren et al., 2013). On the other hand, the response of skin in an extreme environment and the widely used thermal based therapies, such as, laser thermokeratoplasty (LTK), laser-induced interstitial thermotherapy (LITT), laser-induced hyperthermia (HT), interstitial laser

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http://dx.doi.org/10.1016/j.ijengsci.2014.04.014
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photocoagulation therapy (ILP) and Pan-Retinal Photocoagulation (PRP), have received researchers' attention (Aweda et al., 2012; Jha & Narasimhan, 2011; Xu, Seffen, & Lu, 2008), which is related to human health and deserves more research efforts. Of above-discussed topics, the temperature rise may be viewed as the common parameter for determining response amplitude, and it is desirable to obtain the complete figure of temperature distribution (Aweda et al., 2012). However, Fourier's law of heat conduction breaks down in these cases (high heat flux and low temperature) (Cao & Guo, 2007), and lots of generalized heat conduction models were proposed (in this paper, super-dot refers to the derivative with respect to time; comma followed by sub-index denotes the corresponding partial differentiation, and summation convention over repeated sub-indices applies):

- Cattaneo and Vernotte (CV) model (Cattaneo, 1958; Vernotte, 1958): $q_i + \tau\dot{q}_i = -k\theta_{,i}$.
- Green and Naghdi (GN) model (Mallik & Kanoria, 2008; Taheri, Fariborz, & Eslami, 2005): $\dot{q}_i = -(k\dot{\theta}_{,i} + k^*\theta_{,i})$.
- Inertial theory (Joseph & Preziosi, 1989): $\dot{q}_i = -k^*\theta_{,i}$.
- Dual phase lag (DPL) model (Tzou, 1995): $q_i(t+\tau_q) = -k\theta_{,i}(t+\tau_\theta)$.
- Thermomass model (Guo & Hou, 2010):

$$\tau_{TM}\frac{\partial q}{\partial t} - l_{TM}\rho c_E\frac{\partial \theta}{\partial t} + l_{TM}\frac{\partial q}{\partial x} - bk\frac{\partial \theta}{\partial x} + k\frac{\partial \theta}{\partial x} + q = 0$$

where q_i, k, θ, ρ, c_E are heat flux, thermal conductivity, temperature, mass density, and specific heat; τ is the relaxation time of CV model, k^* is the material constant characteristic of GN model, τ_q, τ_θ denote the relaxation time of DPL model, τ_{TM}, l_{TM}, b are relaxation time, characteristic length parameter, and square of thermal Mach number of thermomass model, respectively. A brief introduction of these models is referred to Section 2 in Yu, Tian, and Lu (2013a, 2013b). In these models, the rigid body assumption of conductive medium is enforced. Once the deformation is considered, it comes to the scope of generalized thermo-elastic dynamics, of which the celebrated models contain LS model (Lord & Shulman, 1967), GL model (Green & Lindsay, 1972), and the GN model (Green & Naghdi, 1992, 1993). Meanwhile, some newly proposed theories, such as inertia entropy model proposed by Kuang (2008) and also the thermomass model thermoelasticity model (Wang, Zhang, & Song, 2013), are expected to be widely used.

On a separate front, investigations concerning fractional derivatives and integrals have been increasing, including theoretical studies and numerical methods, such as the existence and uniqueness of solutions of fractional differential equations and corresponding algorithms essential for solving practical problems. In recent years, inspired by fractional calculus's successful application in anomalous diffusion, generalized thermoelasticity models have been further extended in the context of fractional calculus:

- In 2005, by proposing a heat conduction equation based on time-fractional derivative, Povstenko (2004) established the first fractional order generalized thermoelasticity model, which interpolated the classical thermoelasticity and the GN generalized thermoelasticity.
- Based upon LS theory, Youssef (2010), Sherief, El-Sayed, and Abd El-Latief (2010) and Ezzat (2011) established fractional thermoelasticity models in different ways, respectively.
- In 2012, Ezzat, El-Karamany, and Ezzat (2012), Ezzat, Karamany, and Fayik (2012) proposed fractional dual and three-phase-lag thermoelasticity model using fractional Taylor's series.
- In 2013, by introducing fractional calculus into LS, GL, GN, and DPL models, Yu, Tian, and Lu (2013a, 2013b) proposed the unified fractional order generalized thermo-elasticity with electro-magnetic effect and micro-modeling.
- Fractional thermoelasticity theory built upon GN model was introduced by Abbas (2014) in 2014.

All of these theories may be totally referred as time-fractional order generalized thermoelasticity. In addition, some spatial modifications with fractional calculus are also reported. In essence, temporal and spatial fractional calculus are commonly applied to express memory-dependence and nonlocality, respectively. These newly proposed fractional heat conduction (thermoelasticity) models by Sherief et al. (2010), Youssef (2010) and Ezzat (2011) may be viewed as extensions of CV (LS) model with memory-dependent effect. However, it is easily seen that they are not unique, namely, different formulas are proposed by different authors, as a consequence, one may conclude that the fractional order generalized thermoelasticity theories are far from a common agreement and deserves more investigation efforts.

It is well known that fractional calculus is still in infancy, although it has been invented for three centuries and applied widely in science and engineering. Recently, Wang and Li (2011) proposed a memory-dependent derivative (MDD), of which *Time delay* and *Kernel function* can be chosen freely according to the necessity of application, and found it is better than the fractional calculus for reflecting the memory effect. In this paper, we aim at establishing a new memory-dependent generalized thermoelasticity based upon MDD, providing an alternative approach to describe memory-dependence that has been commonly depicted by fractional generalized thermoelasticity. To solve the problem, Laplace Transform is applied, while an efficient algorithm NILT is performed in numerical inverse Laplace Transform. Some parametric studies are performed to evaluate the effect of *Time delay* and *Kernel function* on the thermo-elastic responses. Finally, some concluding remarks are summarized.