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Proposition of a modal filtering method to enhance heat source computation within heterogeneous thermomechanical problems

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A B S T R A C T

This paper presents a new approach to evaluate heat sources from thermal field measurements. A modal projection based on dynamics (Discrete Modal Decomposition) is used to estimate the spatial term of a heat diffusion problem. A numerical example is presented in order to validate this approach and compare it to a more classical spectral decomposition (based on thermal considerations). Results show that the proposed projection basis not only provides closer assessment of the heat sources but is also more stable to noise and side effects. Finally, a basis enrichment method is presented and tested. It shows that *a priori* knowledge of the heat sources shape though approximated (e.g. from strain measurement) enhances the assessment of calorific effects accompanying material deformation.

1. Introduction

The study of calorific effects associated with material deformation has been widely investigated since the early works of Lord Kelvin. In the late 90 s, experimenters have turned to the use of infrared (IR) thermography in order to estimate the temperature variations at the sample surface during mechanical loading. The observed temperature variations depend on both the material and the type of loading and can, under some assumptions, provide valuable data on the material behavior. For example, positive temperature variations in one area of a loaded sample may reveal that irreversibilities take place at this location. However, the intensity of the temperature variations does not only depend on the internal mechanisms of the material (reversible or not). They also depends on various characteristics of the thermal problem (geometry of the studied sample, thermophysical properties, boundary and initial conditions). Within this context, another approach, based on heat sources analysis, was

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proposed and has proved its worth on various thermomechanical problems (Chrysochoos, Maisonneuve, Martin, Caumon, & Chezeaux, 1989; Chrysochoos & Louche, 2000; Louche & Chrysochoos, 2001; Louche, Vacher, & Arrieux, 2005).

Hence, heat sources has become relevant quantities to study material behavior. They provide complementary informations on the instantaneous energy, locally associated with the material behavior. Either the sign and the magnitude of this volumic power (denoted w'_{ch}) can be linked to the material underlying physics and can also be used to achieve energy balance (Chrysochoos, Wattrisse, Muracciole, & El Kaïm, 2009; Dumoulin, Louche, Hopperstad, & Børvik, 2010).

Classically, the heat sources computation relies on performing both spatial and time derivation of the measured temperature fields $T(x,y,t)$. Under some classical assumptions (Chrysochoos & Louche, 2000), the 2D-heat diffusion equation is given by

$$\rho C \frac{\partial \theta}{\partial t} - k\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) = w'_{ch} \quad \text{with } \theta(x,y,t) = T - T_0, \tag{1}$$

where θ stands for the temperature variation, x and y are the spatial coordinates, ρ is the mass density, C the thermal capacity, k the thermal conductivity, $T_0(x,y)$ the initial temperature field and w'_{ch} the internal heat sources. Such a definition allows the assessment of w'_{ch} from the time-derivative and the second space-derivative of $\theta(x,y,t)$ and the *a priori* knowledge of the thermophysical constants ρ, C and k.

In practice, the time resolution of modern IR cameras (up to 380 *Hz*) often allows the computation of a proper time derivative of θ using low-pass or polynomial filters at every pixel. On the other hand, the space-derivative term cannot be estimated as straight-forwardly. The measurement noise, intrinsic to IR-thermography, is amplified by space-derivation operators and massively impairs the calculation of the Laplace terms in Eq. (1).

Many numerical approaches have been developed in the last decade in order to estimate as accurately as possible the left hand side term of Eq. (1). These regularisation methods can be classified into 3 main categories:

- Filtering methods, aimed at suppressing the noise from the measurement using Gaussian (Bozzoli, Pagliarini, & Rainieri, 2013; Xiaoyan, Dongsheng, Yu, Jieyan, & Nie, 2012) or low pass filters based on linear, Fourier or wavelet definitions (Chrysochoos & Louche, 2000; Chuli & Chunyu, 2003; Yi & Murio, 2002).
- Inverse methods, aimed at obtaining a quasi-solution through the use of one or several optimization algorithm (Kaipio & Somersalo, 2007; Wong & Kirby, 1990).
- Projection methods, they are among classical means in inverse heat transfer analysis. They consist of decomposing the measurement within a spectral basis built from eigen function of the Laplace operator (Chrysochoos & Louche, 2000; Doudard, Calloch, Hild, Cugy, & Galtier, 2005). These techniques have recently experienced various improvements through the use of wavelet decompositions (Candau, 2005) or branch modes decomposition (Neveu, El-Khoury, & Flament, 1999). This latter approach allows the use of generalized boundary conditions which usually represents the main disadvantage of projection methods. Mixed temporal (Fourier) and spatial (Laplace) projection methods have also been used and provided promising results (Renault, Andréa, Maillet, & Cunat, 2010). Nevertheless, boundary conditions and heat sources reconstruction on the domain contour remains a significant drawback of such approaches.

The present paper proposes to implement a projection method and compare the results obtained using a decomposition built from structural dynamics and another from thermomechanical considerations. The use of a basis, not related to the solution of a given problem, is nowadays among classical means in noise removal approaches (Adragna, Samper, Pillet, & Favrelière, 2006; Le Goic, Favrelière, Samper, & Formosa, 2011; Wang, Mottershead, Sebastian, & Patterson, 2011). For the sake of comparison, a numerical example is built (Section 3). The heat sources are then rebuilt using *(i)* the proposed modal basis (Section 2.2) and *(ii)* the classical spectral basis (Section 2.3). Hence, the influence of various parameters along with the measurement noise are investigated (Section 4). Finally, a basis enrichment approach is proposed and tested (Section 5).

2. Setup of projection bases

2.1. Projection operator

The thermal field measurement, such as provided by an IR-camera, is a film made of 2-dimensional arrays. The finite resolution of the capture leads to define an integer grid $\mathcal{M} = \mathbb{R}_\delta \times \mathbb{R}_\delta \cap [0;a] \times [0;b]$ which step is denoted δ. In addition, the constant frame rate results in the measured field being part of a monotonous discrete sequence:

$$\theta(x,y) = (\theta_1(x,y); \theta_2(x,y); \ldots; \theta_k(x,y)), \tag{2}$$

where each time step (denoted using subscript $k \in \mathbb{N}^*$) is a temperature variation frame.

In a space continuous framework, the use of the modal/spectral approach requires the projection of the measured fields $\theta_k(x,y) \in \mathbb{R}$ into an eigen basis $\mathfrak{B} \subset \mathcal{C}^2([0;a] \times [0;b], \mathbb{R})$ independent of the step time k. This basis is defined by its continuous eigen vectors denoted $(Q_i^c)_{i\in\mathbb{N}^*}$. Hence, assuming that $\mathfrak{B}$ is a basis of the solution subspace $\mathcal{S}$ of Eq. (1), the measured field can be expressed as a linear combination of the eigen vectors: