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Introduction of external magnetic fields in entropic moment modelling for radiotherapy

J. Page *, Ph. Nicolaï, G. Birindelli, J. Caron, B. Dubroca, G. Kantor, V. Tikhonchuk, J.-L. Feugeas

CELIA Centre des Lasers Intenses et Applications, Talence, France
CRLCC Institut Bergonié, Radiotherapy, Bordeaux, France

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ABSTRACT

One of the big challenges of the emerging MRI-guided radiotherapy is the prediction of an external magnetic field effect on the deposited dose induced by a beam of charged particles. In this paper, we present the results of the implementation of the Lorentz force in the deterministic M1 model. The validation of our code is performed by comparisons with the Monte-Carlo code FLUKA. The relevant examples show a significant modification of the shape of dose deposition volume induced by the external magnetic field in presence of heterogeneities. A gamma-index analysis 3%/3 mm shows a good agreement of our model with FLUKA simulations.

1. Introduction

Radiotherapy is an efficient method for tumor treatment, which consisting of delivering a high dose of energetic particles to a target volume. Numerous complementary techniques such as Intensity Modulated Radiotherapy (IMRT) [1], Volumetric Modulated Arc Therapy (VMAT) [2], and Magnetic Resonance Imaging (MRI)-guided radiotherapy [3] are used to increase the reliability and adaptability of radiotherapy, specifically when the tumors are located near organs at risk. MRI precisely shows the tumor position in real-time and allows directing the photon beams to the tumor while sparing organs at risks. Combining devices in this way improves the treatment precision.

Radiation treatment is prepared with numerical algorithms [4] enabling calculation of the dose deposition as a function of the tumor position and radiation source characteristics. The Pencil Beam algorithms [5] are able to reproduce a particle beam propagation by presenting it as a composition of very thin beams, but may suffer from insufficient accuracy [6,7]. The Collapsed Cone Convolution [8] consists of calculating the dose with pre-calculated kernels. This algorithm is an improvement of the Pencil beam algorithm, and has a better accuracy, specifically in presence of heterogeneities. Monte Carlo (MC) algorithms simulate a beam propagation with a large number of test particles, allowing the calculation of the dose deposition with high precision [9], nevertheless the computation time is too long for clinical treatments. Special techniques allow speeding up the calculation time of MC algorithms [10]. However, the numerical performance and the high noise level remain challenging. Deterministic algorithms [11] offer a low noise and high precision calculation. These algorithms resolve the transport equation on a fixed grid, describing the particle beam as a continuous fluid [12,13]. However, there is no commercially available algorithm in radiotherapy that is able to calculate the modifications of beam propagation and energy deposition due to an external magnetic field. Some studies have already shown the feasibility of including magnetic fields in deterministic [14] and MC algorithms [15–17].

In the present paper we introduce an external magnetic field in the deterministic algorithm M1, that allows simulating the dose deposition in a medium using the linear Boltzmann kinetic equation. It is reduced to two equations for angular moments with an entropy closure in the phase space. It has already been proven that such an approach describes the electron transport with good precision and is suitable for applications in radiotherapy [18]. We present a new version of our algorithm which allows taking into account the influence of the magnetic field on the particle transport via the Lorentz force. We compare our code simulation results with the reference MC code, FLUKA [19,20]. The tests with homogeneous and heterogeneous phantoms, with and without magnetic field demonstrate a good precision and numerical performance of our algorithm.

2. Materials and methods

2.1. M_1 model and the Lorentz force

The transport and energy deposition of energetic electrons in a medium is described by the linear Boltzmann transport equation

* Corresponding author.
E-mail address: jonathan-page@live.com (J. Page).

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(LBTE). In radiotherapy context, it is sufficient to consider a stationary form of this equation, that particles neither generate electric and magnetic fields nor interact with each other, but only with electrons and atoms of the medium. In presence of magnetic fields, the Lorentz force modifies the charged particle trajectories and may affect the dose deposition. The LBTE for electrons, in presence of an external magnetic field, can be rewritten as follows:

$$\begin{aligned}&\overrightarrow{\Omega}\cdot\nabla\Psi(x,\varepsilon,\overrightarrow{\Omega})-\frac{q}{pc}\overrightarrow{\Omega}\wedge\overrightarrow{B}\cdot\nabla_{\overrightarrow{\Omega}}\Psi(x,\varepsilon,\overrightarrow{\Omega})\\&\quad=\rho(x)\int_0^\infty\int_{S_2}\sigma(\varepsilon',\varepsilon,\Omega'\cdot\Omega)\Psi(x,\varepsilon,\overrightarrow{\Omega})d\Omega' d\varepsilon'\\&\qquad-\rho(x)\Psi(x,\varepsilon,\overrightarrow{\Omega})\int_0^\infty\int_{S_2}\sigma(\varepsilon,\varepsilon',\Omega'\cdot\Omega)d\Omega' d\varepsilon'\end{aligned}$$

where Ψ is the electron flux function, which depends on the position x, energy ε and direction Ω. ρ is the mass density of the medium, q is the electron charge, p its momentum, c the speed of light, and B the amplitude of the magnetic field. The cgs unit system is used in this manuscript. The left-hand side of this equation is composed of the advection term and their deflection induced by the Lorentz force. The right-hand side describes the collisional processes: the first term accounts for the gain of number of particles with the energy ε and direction Ω from particles having other energy ε_0 and direction Ω_0; σ is the microscopic differential cross section of elastic and inelastic collisions. The second term accounts for the loss of particles with energy ε and direction Ω in a collision with other particles. Although our model simultaneously treats both electrons and photons, only the equation for electrons is presented here, as the photon transport is not affected by the magnetic fields. However, the simulations were performed with the code for both particles.

The LBTE contains too many variables and is not suitable for radiotherapy applications. It is reduced to two simpler equations for angular momenta with entropic closure [18]. This approach allows us to eliminate angular variables and reduce the calculation time while preserving a good accuracy. The three first moments Ψ_0, Ψ_1 and Ψ_2 of the distribution function are obtained by integrating the LBTE over the unit sphere of the phase space, as follows:

$$\Psi_0(x,\varepsilon)=\int_{S_2}\Psi(x,\varepsilon,\overrightarrow{\Omega})d\overrightarrow{\Omega}$$

$$\overrightarrow{\Psi_1}(x,\varepsilon)=\int_{S_2}\overrightarrow{\Omega}\Psi(x,\varepsilon,\overrightarrow{\Omega})d\overrightarrow{\Omega}$$

$$\overline{\overline{\Psi_2}}(x,\varepsilon)=\int_{S_2}\overrightarrow{\Omega}\overrightarrow{\Omega}\Psi(x,\varepsilon,\overrightarrow{\Omega})d\overrightarrow{\Omega}$$

here, Ψ_0 represents the density of particles per unit energy, Ψ_1 their fluence and Ψ_2 is a tensor which can be interpreted as pressure. These moments depend on the position and energy of particles. The kinetic equations for the two first moments are as follows:

$$\begin{aligned}&\rho(x)\sigma_T(\varepsilon)\Psi_0(x,\varepsilon)+\nabla_x\cdot\overrightarrow{\Psi_1}(x,\varepsilon)\\&\quad=\rho(x)\int_\varepsilon^\infty\sigma^0(\varepsilon',\varepsilon)\Psi_0(x,\varepsilon')d\varepsilon'\end{aligned}\tag{1}$$

$$\begin{aligned}\rho(x)\sigma_T(\varepsilon)\overrightarrow{\Psi_1}(x,\varepsilon)+\nabla_x\cdot\overline{\overline{\Psi_2}}(x,\varepsilon)&=\rho(x)\int_\varepsilon^\infty\sigma^1(\varepsilon',\varepsilon)\overrightarrow{\Psi_1}(x,\varepsilon')d\varepsilon'\\&\quad+\frac{q}{pc}\overrightarrow{\Psi_1}\wedge\overrightarrow{B}\end{aligned}$$

where σT is the total cross section defined as:

$$\sigma_T(\varepsilon)=\int_0^\infty\int_{S_2}\sigma(\varepsilon,\varepsilon',\overrightarrow{\Omega}'\cdot\overrightarrow{\Omega})d\overrightarrow{\Omega}'d\varepsilon'$$

The set of Eq. (1), called M1, is closed by using the approach similar to the Boltzmann's H-Theorem [21]. This is a constitutive part of modern statistical physics which stipulates that a closed system left to itself tends to reach a state of equilibrium characterized by a maximum entropy. The function achieving this condition under the constraint of Ψ_0 and Ψ_1 definition is an exponential distribution function composed of a first order polynomial function of Ω. Then the third moment Ψ_2 can be written as follows [22,23]:

$$\frac{\overline{\overline{\Psi_2}}}{\Psi_0}=\frac{1-\chi(\alpha)}{2}\overline{\overline{Id}}+\frac{3\chi(\alpha)-1}{2}\frac{\overrightarrow{\Psi_1}}{(\overrightarrow{\Psi_1})}\left(\frac{\overrightarrow{\Psi_1}}{(\overrightarrow{\Psi_1})}\right)$$

χ is the Eddington factor dependent on α, which is the anisotropy factor defined as $\alpha = \Psi_1/\Psi_0$. An isotropic distribution is obtained for $\chi = 1/3$ and a mono-directional beam is obtained for $\chi = 1$. The expression of this factor defines the closure of the system. The major simplifying hypothesis of this approach is that the underlying distribution function does not depend explicitly on the magnetic field, but only implicitly through the first moment function, Ψ_1.

The M1 model (2) is solved on a three-dimensional grid composed of voxels. The energy distribution is described with a discrete number of groups and the dose deposition is calculated in each of voxels. The numerical scheme is described in [24]. It was shown in [18] that this model has accuracy comparable to the reference MC codes while being much less time and memory consuming.

In the present work, we demonstrate the performance of the M1 model with the magnetic field included in the moment Eq. (1) by comparing the results with the reference MC code FLUKA (FLUktuierende KAskade). The magnetic field appears in the equation for Ψ_1 and is responsible for modification of the particle propagation: it induces a deflection of particles from their original direction orthogonal to the fluence and the orientation of the magnetic field. The introduction of this term is motivated by the recent requirements for dedicated new MRI-guided radiotherapy facilities.

2.2. Monte Carlo reference code

The dose depositions calculated with the entropic model (2) are compared with the reference simulations conducted with the FLUKA code [19,20]. This full MC takes into account the external electric and magnetic field effects. In the present simulations the size of the mesh grid is 1 mm^3. The simulations were run with 1×10^8 particles allowing reasonable reduction of the statistical noise level. The minimum energy was fixed at 10 keV for all calculations.

2.3. Test cases

The results of simulations with the two codes were compared using electron and photon beams, with or without the magnetic field. The simulations were conducted on a laptop running on eight Intel® Core™ i7-4710MQ CPUs @ 2.50 GHz cores. We simulated the propagation of a monoenergetic 6 MeV electron beam, of field size 10×45 mm^2, in a $45 \times 45 \times 35$ mm^3 water phantom (Fig. 1), with a 1 T magnetic field directed perpendicular to the beam axis. We also simulated a monoenergetic 6 MeV photon beam of field size 60×100 mm^2 propagating through two different phantoms. The first one was a homogeneous $100 \times 100 \times 200$ mm^3 water phantom and the second one was heterogeneous, composed of a lung insert (density = 0.3 g/cm^3) of dimension $100 \times 100 \times 100$ mm^3 between two water pieces, of $100 \times 100 \times 50$ mm^3 each (Fig. 2). In both cases, when activated, the magnetic field of 1 T was directed orthogonally to the propagation of the beam.

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