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Particle-free bodies and point-free spaces ☆

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ABSTRACT

Few notions in mathematics and physics are as fundamental and useful as the notion of a "point". However, in addition to the concept of a "point" being far from apparent, the concept is not suitable for describing several important problems in natural philosophy. A far more tangible and sensible idea that is immediately grasped by our mind is that of a "chunk" (a solid object) which seems ideally suited to describe precisely those problems which the notion of "point" seems to hinder. In this paper, I articulate the need for the use of topologies which are "point free" for the proper resolution of many important problems in natural philosophy.

1. Introductory remarks

I resisted the temptation to try and appear droll and title this work "Pointless bodies" as I remembered reading the witticism "A celebrated reviewer once described a certain paper (in a phrase which never actually saw publication in Mathematical Reviews) as being concerned with the study of 'valueless measures on pointless spaces" in Johnstone's article titled "The point of pointless bodies" (see Johnstone, 1983); I was apprehensive that this paper might suffer a review which actually sees publication that expresses the sentiment that it is a "pointless paper on pointless bodies".

The subject matter of this paper, namely what one means by a body and the space that it occupies and how one mathematically models it, has been the object of study from ancient to current time without any consensus with regard to the answer to these questions. These questions have attracted the attention and interest of philosophers concerned primarily with metaphysical issues, logicians interested in the foundations of natural philosophy, rigorous topologists and analysts, physicists, and engineers, the conclusions of the various members of the group resembling Saxe's rendition of the Indian tale of the six blind Indians describing the form of the elephant. Even a cursory glance at the various writings on the subject, papers, review articles and books, reveals the insular interests of the members of the different groups that seem to be either ignorant of the interests and investigations of the other groups, or if aware obviously dismissive of the views of the other groups. I guess one should not be astonished at such ignorance amongst specialist scholars as this is true of even the best amongst us as evidenced by Collingwood's (1986) remarks that as distinguished a philosopher as Whitehead was unaware of some of the most important works of Aristotle; such ignorance may have

☆ This paper delves into the notion of a "point" as it pertains to its usage in mathematics; it, whatever "it" is has "*position but no magnitude*". However, the word is also used in another sense, namely as "*essential or indispensable*". This paper is dedicated to Chandrika who is the "point" to my life, in this latter sense of its usage.

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been all for the good as oftentimes pronouncements and writings of authorities can have a chilling effect on creativity and the creation of knowledge.[1]

Before we get into a discussion of what one means by a body in continuum mechanics, which is the main thrust of this work, it is necessary to start with a discussion of the simplest model for a body that was used in Newtonian mechanics, namely the notion of a body in Newtonian Particle Mechanics. Within the construct of Particle Mechanics, a body is mathematically idealized to be a "point mass", that is it is assumed that a body can be represented as a "point"[2] mathematically, with the "point" having a finite mass.

The notion of a "particle" in physics, and a "point" in mathematics are idealizations, the former is supposedly an approximation of "reality" in physics and the latter a conceptualization of a primitive variable in geometry. Both these notions have served their respective fields so well that the scientific temper of a person that suggests that the applicability and usefulness of the concepts be examined, to be more accurate re-examined and carefully delineated, would be called into question. This notwithstanding, even at the cost of being ridiculed, in this short paper I argue that the applicability of the concept of "point", as it pertains to the problems of natural philosophy needs to be reassessed, especially in light of our inability to address many important issues within such a construct. In fact, the notion as it is conceived today is an impediment in the proper resolution of a variety of problems in natural philosophy, and I shall discuss these problems in the course of this article.

To ask the question "why" seems to be a compelling trait of the human mind. We are not satisfied by being merely aware of "facts", there seems to be a compulsion to enquire as to why it is indeed a fact. We pride ourselves that as a species we are unique in possessing this trait of asking the question "why" and not merely "doing and dying" and that this is the characteristic that distinguishes us from a trained Chimpanzee.[3]

Whether it is classical continuum mechanics or more modern theories of physics like Quantum Mechanics,[4] one has to deal with the notion of a body, and the equations that govern the motion of matter are given through field equations.[5] Since we are defining field equations that hold at every "point" in the space of interest, we are dealing with equations that apply to "points" belonging to a certain space (in classical mechanics) or space–time (in more modern aspects of physics). It is far from clear that one can in fact specify field equations for the quantities that we introduce in physics as matter does not occupy every point in space wherein the body seems to lie as far as the naked eye is concerned. In Quantum Mechanics, "probabilities" are associated with the physical variables that appear in the governing equations. Also, there is supposedly an "uncertainty" with regard to the precision with which certain pair of physical quantities, such as position and momentum, can be determined simultaneously. Whether there is "uncertainty" with regard to simultaneously knowing physical quantities with precision, and whether occurrence of natural phenomena need to be prescribed in a probabilistic manner is an issue that needs some discussion. Whether it is our ignorance[6] of the precise makeup of the natural world that is the ground spring for the apparent lack of certainty that prompts us to provide a probabilistic description for them and the laws governing them is an unsettled question despite the legions that subscribe to the conventional wisdom of Quantum Mechanics. While most physicists cling to the point of view that natural phenomena are inherently random and non-deterministic, holding up the successes of their theories as validating and justifying them, the point of view for a more deterministic approach to Quantum Mechanics, which I subscribe to, is well articulated by the likes of Bohm (see Bohm (1957), Bohm and Hilley (1993)). If indeed there is no randomness or uncertainty[7] with regard to the occurrence of events, our assigning a probability and then carrying out a set of deductions based on a set of rules is no different than devising a devilishly clever game; and the game however demanding of one's intelligence is no more than a game. David Hilbert (see Rose (1988)) is supposed to have said "Mathematics is a game played according to certain simple rules with meaningless marks on paper".[8]

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[1] Roger Bacon's (Bacon, 1267) following warning concerning the impediments to our grasping the truth is most appropriate: *"Now, there are four chief obstacles in grasping truth, which hinder everyman, however, learned, and scarcely allow anyone to win a clear title to learning, namely submission to faulty and unworthy authority, influence of custom, popular prejudice and the concealment of our own ignorance accompanied by an ostentatious display of our knowledge"*.

[2] We need to recognize that there are two questions that need investigation and need to be distinguished during the course of this article. The first question is concerned with what we mean by a "point" and the second questions concerns whether such a notion can be used in the construction of extended solid objects.

[3] I am not too sure that this is indeed the case; have the Chimpanzees figured out that asking the question "why" is interminable and that human beings are a lesser life form for not having recognized this "fact"?

[4] It would not be unreasonable to question the validity of Quantum Mechanics as propounded by the Copenhagen school. While the proponents of Quantum Mechanics could point to the fact that its predictions are in agreement with numerous experimentally observed results, it is important to bear in mind that such successful comparisons concern very simple systems. To date, Quantum Mechanics has not been used to predict phenomena concerning sufficiently complex systems of molecules.

[5] Clifford (1882) suggested a theory of matter wherein the movement of matter through space is the propagation of the variation of the curvature of space.

[6] Probabilities ascribed to quantities in physics are no more than "indices of ignorance". While it might sound anachronistic to espouse the views of Laplace (1814), namely: *"We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes"*, at this juncture in time, I feel that if we do not identify the "intellect" that Laplace refers to with Laplace's Demon or Laplace's Superhuman, but merely recognize that science at some point in time might make it possible for any scientist to be aware of all the information that is necessary to make what is deemed to be random now, deterministic, then I feel that Laplace's sentiments are definitely as sensible a point of view , if not a more sensible view than that held by the Copenhagen School.

[7] The deterministic point of view has its own proponents; the oft quoted comment "God does not play dice" by Einstein is not Einstein expressing his religious faith, his intent was definitely "Nature does not play dice".

[8] Of course, mathematics is much more than merely a game. I believe mathematics to be a language, the best language in which to express the secrets of nature. The great scientist Josiah Willard Gibbs is supposed to have said at a faculty meeting at Yale during a discussion on whether more time should be spent learning foreign languages at the expense of mathematics that *"Mathematics is a language"*. We need to use the right terminology to express the secrets that nature holds.