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Size dependent buckling analysis of microbeams based on modified couple stress theory with high order theories and general boundary conditions

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ABSTRACT

In this research, buckling analysis of three microbeam models are investigated based on modified couple stress theory. Using Euler–Bernoulli beam theory (EBT), Timoshenko beam theory (TBT) and Reddy beam theory (RBT), the effect of shear deformation is presented. To examine the effect of boundary condition, three kinds of boundary conditions i.e. hinged–hinged, clamped–hinged and clamped–clamped boundary conditions, are considered. These nonclassical microbeam models incorporated with Poisson effect, contain a material length scale parameter and can capture the size effect. These models can degenerate into the Classical models if the material length scale parameter and Poisson's ratio are both taken to be zero. Governing equations and boundary conditions are derived by using principle of minimum potential energy. Generalized differential quadrature (GDQ) method is employed to solve the governing differential equations. Also an analytical solution is applied to determine the critical buckling load of microbeams with hinged–hinged boundary condition. Comparison between the results of GDQ and analytical methods reveals the accuracy of GDQ method. Some numerical results are exhibited to indicate the influences of beam thickness, material length scale parameter and Poisson's ratio on the critical buckling load of these microbeams.

1. Introduction

Microbeams, the beams whose characteristic sizes down to the order of micron and sub-micron, have been widely used in micro- and nano-electro-mechanical systems (MEMS and NEMS) (Batra, Porfiri, & Spinello, 2008; Coutu, Kladitis, Starman, & Reid, 2004; Fu & Zhang, 2010; Hua et al., 2007; Li, Bhushan, Takashima, Baek, & Kim, 2003; McMahan & Castleman, 2004; Mahdavi, Farshidianfar, Tahani, Mahdavi, & Dalir, 2008; Pei, Tian, & Thundat, 2004). Size dependent behaviors have been experimentally observed in small-scale structures (Fleck, Muller, Ashby, & Hutchinson, 1994; Lam, Yang, Chong, Wang, & Tong, 2003; McFarland & Colton, 2005; Stolken & Evans, 1998).

Since the beam models based on Classical continuum theories are not capable of describing such size dependent behaviors in micro-scale elements, several nonclassical continuum theories such as the nonlocal, strain gradient and couple stress theories that contain additional material length scale parameters have been developed to capture the size effect.

In the micropolar theory developed by Eringen et al. (1968), Eringen (2002), the stress at a point is a function of strains at all points in the continuum body. Peddieson, Buchanan, and McNitt (2003) applied stated theory to study the static analysis of Euler–Bernoulli beam for clamped–hinged boundary condition. The nonlocal Euler–Bernoulli, Timoshenko, Reddy and

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Levinson beam models formulated by Reddy (2007) and obtained equations were solved analytically. Using meshless method, bending, buckling and free vibration of Timoshenko nanobeams based on nonlocal theory for hinged–hinged boundary condition are studied by Roque, Ferreira, and Reddy (2011). Using the stated theory, Thai (2012) investigated the bending, buckling and vibration of simply supported nanobeams.

Mindlin (1965) and Mindlin and Eshel (1968) introduced a higher order gradient theory for elastic materials. Fleck and Hutchinson (1997) extended and reformulated the mentioned theory and called as the strain gradient theory that contains five higher order material length scale parameters in addition to the two Lame constants. Considering a new additional equilibrium equation, Lam, Yang, Chong, Wang, and Tong (2003) developed a modified strain gradient elasticity theory that contains three higher order material length scale parameters in addition to the two Lame constants for isotropic linear elastic materials. Kong, Zhou, Nie, and Wang (2009) utilized the stated theory to study the static and dynamic behaviors of linear Euler–Bernoulli microbeams. Wang, Zhao, and Zhou (2010) investigated the behavior of linear Timoshenko microbeams. Kahrobaiyan, Asghari, Rahaeifard, and Ahmadian (2011) presented the static and dynamic behaviors of nonlinear Euler–Bernoulli microbeams for hinged–hinged boundary condition. Static and free vibration of size-dependent functionally graded Euler–Bernoulli beam model are studied by Kahrobaiyan, Rahaeifard, Tajalli, and Ahmadian (2012) and obtained equations were solved analytically. Using stated theory, Rahaeifard, Kahrobaiyan, Ahmadian, and Firoozbakhsh (2013) studied the size-dependent static and dynamic behavior of nonlinear functionally graded Euler–Bernoulli beam model for hinged–hinged boundary condition. Akgoz and Civalek (2013) developed a size-dependent higher-order shear deformation beam model based on mentioned theory and studied static bending and free vibration behavior of simply supported microbeams.

In 1960s, the couple stress theory that contains two higher order material length scale parameters in addition to the two Lame constants is proposed by Mindlin and Tiersten (1962), Toupin (1962), Mindlin (1964) and Koiter (1964). Employing the Classical couple stress theory, Anthoine (2000) studied the pure bending of circular cylinder. Using a new additional equilibrium equation, the equilibrium of moments of couples, in addition to the classical equilibrium equations of forces and moments of forces, Yang, Chong, Lam, and Tong (2002) developed a modified couple stress theory. One of the good aspects of this theory was that the additional parameters were reduced to one additional parameter. Utilizing the modified couple stress theory, Park and Gao (2006) and Kong, Zhou, Nie, and Wang (2008) investigated the static and dynamic behaviors of linear Euler–Bernoulli microbeams, respectively. Ma, Gao, and Reddy (2008, 2010) employed the stated theory to study the static bending and free vibration of microbeams for hinged–hinged boundary condition using Timoshenko and Reddy-Levinson beam theories, respectively. Ke, Wang, and Wang (2011) determined thermal free vibration and buckling of nonclassical Timoshenko beams for hinged–hinged boundary condition based on modified couple stress theory. Free vibration and buckling of micro FG beams using Timoshenko beam theory were studied by Ke and Wang (2010). Asghari, Kahrobaiyan, and Ahmadian (2010) investigated the static bending and free vibration behavior of size dependent nonlinear Timoshenko beam under constant transverse distributed force for hinged–hinged boundary condition and solved the equations by finite difference method. Xia, Wang, and Yin (2010) investigated the size effect on the nonlinear bending, nonlinear vibration and postbuckling of Euler–Bernoulli beam for hinged–hinged boundary condition. Reddy (2011) utilized the modified couple stress theory to study the bending, buckling and free vibration of nonlinear micro FG beams for hinged–hinged boundary condition considering Euler–Bernoulli and Timoshenko beam theories. Employing modified couple stress theory and strain gradient elasticity, Akgoz and Civalek (2011) studied the buckling problem of Euler–Bernoulli micro beams for hinged–hinged and clamped–hinged boundary conditions. Nonlinear free vibration of micro functionally graded Timoshenko beam for hinged–hinged boundary condition is investigated by Ke, Wang, Yang, and Kitipornchai (2012) and solved the equations by differential quadrature method. Simsek and Reddy (2013) studied the static bending and free vibration of simply supported functionally graded microbeams based on stated theory and various higher order beam theories. Asghari (2012) studied the nonlinear micro-plates with arbitrary shapes based on the modified couple stress theory.

In this paper buckling analysis of microbeams are investigated based on modified couple stress theory using a variational formulation that is based on principle of minimum potential energy. To present the effect of shear correction factor three beam theories i.e. EBT, TBT and RBT are considered. Using hinged–hinged (h–h), clamped–hinged (c–h) and clamped–clamped (c–c) boundary conditions, effect of boundary conditions is studied. The modified couple stress theory contains a material length scale parameter to capture the size effect. These models can degenerate into the Classical models if the material length scale parameter and Poisson's ratio are both taken to be zero. Using generalized differential quadrature (GDQ) method, critical buckling loads of these microbeam models are obtained. Also, the obtained differential equations are analytically solved for hinged–hinged boundary condition. Comparison between the results of GDQ and analytical methods reveals the accuracy of GDQ method. At the end some numerical results are presented to study the effects of material length scale parameter, beam thickness and Poisson's ratio on the microbeam behavior.

2. Equations modeling

According to modified couple stress theory elaborated by Yang et al. (2002), the strain energy U in a deformed isotropic linear elastic material occupying region Ω is given by:

$$U = \frac{1}{2}\int_{\Omega}(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij})dv - \frac{1}{2}\int_0^L \left(P_b\left(\frac{\partial w}{\partial x}\right)^2\right)dx \quad i,j = 1,2,3 \tag{1}$$