International Journal of Engineering Science 74 (2014) 55–64

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Thermorheological effect on thermal nonequilibrium porous convection with heat generation

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ARTICLE INFO

Article history:
Received 25 April 2013
Received in revised form 31 July 2013
Accepted 16 August 2013
Available online 25 September 2013

Keywords:
Porous medium
Thermorheology
Thermal nonequilibrium
Brinkman's equation

ABSTRACT

Thermorheological effect on the sufficient conditions for the onset of natural convection in a fluid saturated porous medium with uniformly distributed internal heat sources is studied. The flow through the medium is governed by Brinkman's equation. The constituent phases of the medium are assumed lack thermal equilibrium. A nonlinear temperature dependent viscosity is considered. The resulting eigenvalue problem is solved using the Galerkin weighted residual method. Results indicate that the temperature dependence of viscosity and the presence of heat sources produce effects that are favourable for convection to set in. A possible application of this study in a practical situation is highlighted.

1. Introduction

Problems in fluid mechanics involving the onset of convection in porous media are of continued interest in order to resolve the intricacies involved in several technological operations such as underground disposal of nuclear waste, petroleum reservoir operations, casting and welding in manufacturing process and chemical and food processing. In particular convection currents in porous media containing internally distributed heat sources are of great importance in geophysical applications, synthesis reactions of several chemicals, etc. Many studies on convective instabilities in porous media in the presence of internal heat sources with or without mass flux are available in the open literature (see for example Rudraiah, Veerappa, and Balachandra Rao (1982), Yoon, Kim, and Choi (1998), Nouri-Borujerdi, Noghrehabadi, and Rees (2007) and Saravanan (2009)).

Most of the works addressing convective instabilities in porous media have invoked Boussinesq approximation. This is not true in some situations, especially when the operating temperature of the system rises beyond a certain level. In particular the viscosity of many liquids is a strong decreasing function of temperature. For example the viscosity of the liquid nitroethane decreases sharply from 1.354 mPa s to 0.337 mPa s when the temperature increases from 248.15 K to 373.15 K at atmospheric pressure. Hence the variation in viscosity was taken into account and its effect on convection in porous media was discussed as early as 1950s by Rogers and Morrison (1950). Over the past few decades a number of authors have considered this effect (see for example Kassoy and Zebib (1975), Patil and Vaidyanathan (1982), Richardson and Straughan (1993), Nield (1996), Saravanan and Kandaswamy (2004), Hooman and Gurgenci (2008a), Shivakumara, Mamatha, and Ravisha (2010), Vanishree and Siddheshwar (2010) and Rajagopal, Saccomandi, and Vergori (2010)).

The works addressing variable viscosity in addition to internal heat generation are limited. Postelnicu, Grosan, and Pop (2001) have analyzed forced convective flow of a internally heat generating and variable viscosity fluid over a flat plate embedded in a porous medium. Bagai (2004) has analyzed the effect of the variable viscosity as well as the internal heat generation on the steady free convection boundary layers over a non-isothermal axisymmetric body embedded in a fluid

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http://dx.doi.org/10.1016/j.ijengsci.2013.08.004

saturated porous medium. He concluded that the viscosity variation contributes a pronounced effect on the velocity profile and the heat transferred is more for a less viscous fluid. Saravanan and Kandaswamy (2004) have studied the stability of convective motion of a variable viscosity fluid contained in a vertical layer generated purely by uniformly distributed internal heat sources in the presence of a transverse magnetic field. It was found that thermal running waves are the most unstable modes and dominate the shear ones when the viscosity decreases.

Studies available on convection in porous media are usually based on thermal equilibrium between the solid and the fluid phases. Nevertheless this assumption cannot be used when their temperatures deviate significantly in situations such as the one involving sudden or high speed convective flows. In such situations the hot fluid stream can penetrate into the porous bed and hence in a representative elementary volume its temperature becomes sufficiently higher than that of the adjacent solid phase. This necessitates to consider the heat transfer characteristics of the fluid and solid phases separately. This Local thermal nonequilibrium (LTNE) situation was discussed in the pioneering work of Combarnous (1972). It was then discussed elaborately by the Rees and his coworkers (see Banu and Rees (2002) and Rees and Pop (2005)) using the two equation model proposed by Nield and Bejan (1999). It was concluded that a thermal shock wave is formed within the fluid phase when the velocity of the fluid is sufficiently large. The presence of heat transfer between the phases caused the strength of the thermal shock to degrade with time. Following this several investigations have been done employing the LTNE model (see for example, Shivakumara et al. (2010), Nield and Kuznetsov (2010) and Saravanan and Jegajothi (2010) and the references cited therein).

The objective of the present paper is to report the onset of natural convection in a porous medium heated from below that is saturated with a heat generating fluid exhibiting temperature sensitive viscosity. Most of the earlier studies have used the classical Darcy's law. It does not capture the flow structure near the bounding rigid surfaces where close packing of the porous fillings is not possible. It is silent about the boundary effects that could become increasingly significant when the porous medium is made up of coarser fillings. Hence we shall employ Brinkman's correction to the Darcy's law which incorporates a Laplacian term analogous to that appearing in the Navier–Stokes equations. According to Rajagopal (2007) the equations due to Darcy and Brinkman can be obtained systematically by making severe approximations on a system of more general constitutive equations which are based on the mixture theory. Brinkman's correction can take care of the boundary effects and has been successfully used in recent studies dealing with convection in complex fluids (see Nield and Kuznetsov (2010) and Sunil, Sharma, and Mahajan (2011)). In addition the porous medium is assumed to be in LTNE state. We also consider the porous medium to be confined between two rigid boundaries, a more realistic situation.

We believe that the results of this study can provide a theoretical support for understanding various processes taking place in packed bed reactors, heat exchangers, etc. (see Levenspiel (1999)). In particular packed bed reactors, which are used in chemical and petroleum industries, contain basically a tube filled with randomly arranged catalytic pellets through which fuels are fed with high speed at various temperatures. Coarser pellets are normally used in order to have high surface area which could speed up the reaction rate. Several exothermic oxidation and hydrogenation processes are usually conducted in these reactors. For example the organic compound isopropanol is synthesized in these reactors by hydrogenating acetone. In this process a stream of acetone which remains in the liquid state enters the tube and releases heat during hydrogenation. One should notice that the viscosity of acetone drops to less than one half its value for an increase in its temperature by 75 K (see Table 2). This specific example corrresponds to a situation in which all the effects to be discussed in this work, viz., Brinkman's extension, LTNE, internal heating and viscosity variation are simultaneously present.

2. Mathematical formulation

We consider a horizontal fluid saturated porous layer of height d which is heated from below and cooled from above. The lower and the upper surfaces of the layer are held at temperatures T_l and $T_u(< T_l)$ respectively. The porous medium is isotropic and homogeneous and is of infinite horizontal extent. It exhibits LTNE and hence we shall employ a two-field model for the energy exchange. However it is assumed that at the bounding surfaces the solid and fluid phases have identical temperatures. Both the two phases of the layer generate heat at a uniform rate q'''. We shall employ Brinkman's correction which has a Laplacian term analogous to that appearing in the Navier–Stokes equations. To describe the geometry we choose a Cartesian coordinate system with its origin at the lower surface and z axis pointing vertically upwards. The viscosity μ of the fluid is assumed to depend on temperature. A nonlinear approximation characterizing as a function of temperature yields the thermorheological equation of state in the form

$$\mu = \mu_0\left[1 - M_1^*(T_f - T_u) - M_2^*(T_f - T_u)^2\right] \tag{1}$$

where M_1^* and M_2^* are empirical constants. The thermal conductivity is treated as a constant as it so for almost all liquids.

The complete system of dimensional conservation equations relevant to the above situation is

$$\nabla \cdot \vec{q} = 0 \tag{2}$$

$$\mu \vec{q} + K(\nabla p - \rho \vec{g}) - K\mu_e \nabla^2 \vec{q} = 0 \tag{3}$$

$$\epsilon(\rho c)_f \frac{\partial T_f}{\partial t} + (\rho c)_f(\vec{q} \cdot \nabla)T_f = \epsilon k_f \nabla^2 T_f + h(T_s - T_f) + \epsilon q_f''' \tag{4}$$