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A new technique for finite element limit-analysis of Hill materials, with an application to the assessment of criteria for anisotropic plastic porous solids

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ABSTRACT

The present work is devoted to the numerical limit-analysis of Hill materials with particular emphasis on anisotropically plastic porous solids. Its aim is to provide an efficient method of limit-analysis based on the standard finite element method including elasticity, and present a few applications.

We first present the numerical implementation of Hill's criterion. We then describe the procedure used for the numerical limit-analysis, which basically consists of using a single large load step ensuring that the limit-load is reached, without updating the geometry. Also, the convergence of the elasto–plastic iterations is accelerated by suitably adjusting the elastic properties of the material.

The method is applied to assess Gurson-like criteria for orthotropically plastic materials containing spheroidal voids. This is done by performing numerical limit-analyses of elementary cells made of a Hill material and containing confocal spheroidal voids, subjected to classical conditions of homogeneous boundary strain rate. The numerical results are compared to the model predictions for both the yield surface and the flow rule, and this permits to discuss the accuracy of the theoretical models considered.

1. Introduction

The most classical model of plastic porous solids, proposed by Gurson (1977), has been obtained by combining homogenization theory and limit-analysis of a spherical (or cylindrical) cell, made of a rigid-ideal-plastic von Mises material containing a spherical (or cylindrical) void, and loaded arbitrarily through conditions of homogeneous boundary strain rate.

Owing to the intrinsic limitation of this model to isotropic materials containing spherical (or cylindrical) voids, several extensions were developed by accounting for void shape effects: Gologanu, Leblond, and Devaux (1993, 1994, 1997)'s model applies to isotropic materials containing spheroidal prolate or oblate voids[1] and is known as the *GLD model*. Some generalization of these works has been very recently proposed by Madou and Leblond (2012a, 2012b) who consider a general ellipsoidal void embedded in a von Mises matrix.

Another type of extension of the Gurson model has consisted in the consideration of anisotropic Hill matrices (Hill, 1948) containing first spherical voids (Benzerga & Besson, 2001), then spheroidal ones (Monchiet, Cazacu, Charkaluk, & Kondo, 2008). Later, Keralavarma and Benzerga (2010) investigated the same problem by considering a richer description of the velocity field.

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[1] Monchiet, Charkaluk, and Kondo (2013) used more sophisticated Eshelby-like velocity fields for the same kind of extension.

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The utility of the models just mentioned lies in the fact that non-spherical voids and plastically anisotropic matrices are quite common in practice.

All these criteria need to be critically assessed through numerical analyses of the representative cells considered in their derivation. While the criteria for isotropic von Mises matrices have already been widely assessed (Gologanu, 1997; Madou & Leblond, 2012b, 2013), those for anisotropic Hill matrices have been studied only in a partial way. Thus, Pastor, Pastor, and Kondo (2012) very recently provided numerical upper and lower bounds of the yield surfaces in the case of spherical and prolate spheroidal cavities, but no results were provided in this work about the flow rules associated to the yield surfaces.

In order to fully understand the effect of plastic anisotropy and void shape, it is necessary to complete Pastor et al. (2012)'s numerical results for spherical, cylindrical, spheroidal prolate and oblate voids embedded in a Hill material, by performing numerical limit-analyses of the representative cells considered. In the present work, we shall consider both the yield criterion and the macroscopic associated flow rule.

The assessment of the models dealing with orthotropic Hill matrices is particularly important given that they are based on a seemingly crude approximation: namely, they consider trial velocity fields identical to those commonly considered in the case of von Mises matrices. Thus, Benzerga and Besson (2001) used the same velocity fields as Gurson (1977) and Monchiet et al. (2008) used those of Gologanu et al. (1993, 1994) and finally Keralavarma and Benzerga (2010) used the richer velocity fields of Gologanu et al. (1997).

Estimated solutions of a limit-analysis problem can be obtained by two complementary approaches providing approximate upper and lower bound solutions, known respectively as the *kinematic and static bounds*. These approaches require some minimization in the kinematic approach and some maximization in the static approach, which can only be achieved by numerical methods in general.

The first attempt to provide numerical solutions of limit-analysis problems was based on linear programming (Anderheggen & Knöpfel, 1972; Bottero, Negre, Pastor, & Turgeman, 1980) and the finite element method: the yield surface was approximately replaced by a polygonal surface, leading to a set of linear inequalities which were solved using linear programming. Both lower and upper bounds have been investigated (Sloan, 1988, 1989; Sloan & Kleeman, 1995). Despite the relatively simplicity of the method, this kind of approach introduces inherent source of inaccuracies in the approximation of the yield surface, and also requires a specific implementation into a finite element code in order to solve the linear system.

Other significant contributions in the field of numerical limit-analysis were the so-called non linear programming methods (Capsoni & Corradi, 1997; Lyamin & Sloan, 2002a; Lyamin & Sloan, 2002b; Krabbenhoft & Damkilde, 2003). With these methods, the limit-analysis problem was directly solved without any linearisation, using non linear algorithms, for instance conic programming or interior point optimization. These works mainly considered isotropic materials governed by von Mises's criterion, and sometimes Drucker–Prager's criterion. Extensions to anisotropic criteria have also been proposed (Corradi, Luzzi, & Vena, 2006; Pastor et al., 2012). These "non linear" methods lead to an estimation of the limit-load without introducing any approximation on the yield surface, but they require complex numerical developments for the optimization procedure.

Recently, Madou and Leblond (2012b) proposed a numerical method to solve limit-analysis problems for von Mises materials, based on the standard finite element method including elasticity, without introducing any approximation of the yield surface or supplementary optimization procedure.

The aim of this paper is to provide a new technique to solve limit-analysis problems for plastically anisotropic materials obeying Hill (1948)'s criterion, based on an extension of Madou and Leblond (2012b)'s procedure for isotropic material. The main advantages of the method proposed are that since it relies on the usual finite element method including elasticity, it requires only few developments, and benefits from the entire know-how developed for, and incorporated in standard codes. (The accuracy of the numerical results it provides has thus been established by Madou & Leblond (2012b), in the case of isotropic materials).

The method will be used to provide numerical results for anisotropic plastic porous materials. These results will be used to assess the models mentioned previously for anisotropic matrices containing spheroidal voids. They could also potentially be used to validate any similar yield criterion.

The paper is organized as follows:

- In Section 2, we first develop a numerical implementation of Hill's criterion and the associated flow rule into a finite element code, suggested by De Borst and Feenstra (1990), in which the sole unknown in the local projection algorithm is the plastic multiplier. A new finite element technique for numerical limit-analysis is then proposed. The principle of this technique consists of using this algorithm with a single large load step, without any geometry update. Also, the elastic properties of the material are suitably adjusted in order to accelerate the convergence of the global elasto–plastic iterations.
- Section 3 investigates the application of this technique to porous plastic materials obeying Hill's criterion. Spheroidal cells containing confocal spheroidal voids are subjected to conditions of homogeneous boundary strain rate. The numerical yield surfaces and flow rules are compared to those predicted by existing models in various cases. This permits to discuss the pertinence of the trial velocity fields used in the derivation of the models.