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The analysis of a mode I conducting crack under general applied loads in piezo-electro-magneto-elastic layer

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A B S T R A C T

Within the theory of linear magnetoelectroelasticity, the fracture analysis of a magneto-electrically limitedly permeable crack embedded in a magnetoelectroelastic layer is investigated. The prescribed normal stress and two cases of electromagnetic boundary conditions on the layer surfaces are adopted. Applying the Hankel transform technique, the boundary-value problem is reduced to solving three coupling Fredholm integral equations of second kind. These equations are solved exactly. The corresponding semi-permeable crack-face magnetoelectric boundary conditions are adopted and the electric displacement and magnetic induction of crack interior are obtained explicitly. This field inside the crack is dependent on the material properties, applied loadings, the dielectric permittivity and magnetic permeability of crack interior, and the ratio of the crack length and the layer thickness. Field intensity factors are obtained as explicit expressions.

1. Introduction

Material having magnetoelectroelastic coupling effects have found increasing applications in engineering structures, particularly in smart materials intelligent structures. The effects of magnetoelectromechanical coupling have been observed in single-phase materials where simultaneous magnetic and electric ordering coexists and in two-phase composites where the participating phases are piezoelectric and piezomagnetic. These "smart" materials are extensively used as electric packaging, sensors and actuators, magnetic field probes, acoustic and ultrasonic devices, hydrophones and transducers with the responsibility of electromagnetomechanical energy conversion. When subjected to mechanical, magnetic and electrical loads in service, these magnetoelectroelastic composites can fail prematurely due to some defects, namely cracks, holes and others, arising during their manufacturing processes. Therefore, it is of great importance to study the magnetoelectroelastic interaction and fracture behaviors of magnetoelectroelastic materials. On the other hand, composites consisting of piezoelectric and piezomagnetic components have found their ways increasingly in applications in engineering structures. This is because these composites possess some new properties of magnetoelectricity with the secondary piezoelectric effects which are not found in single-phase piezoelectric or piezomagnetic materials. In some cases, the magnetoelectric effect of piezoelectric/piezomagnetic composites can be obtained by a hundred times longer than that of a single-phase magnetoelectric material. Recently, Chen, Lee, and Ding (2004) derived general solution for transversely isotropic electromagnetothermoelastic material. In consequence the components of coupled field are expressed by five mono-harmonic functions. More recently, the penny-shaped crack in magnetoelectroelastic material has been considered. For example, Zhao, Yang, and Liu (2006) analyzed a penny-shaped crack in a magnetoelectroelastic medium. Niraula and Wang (2006) derived an exact closed-form solution for a penny-shaped crack in a magnetoelectrothermoelastic material in a temperature field. The electro-magnetic field inside the crack was taken into account and closed form solutions were derived for impermeable and permeable crack

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Nomenclature

a	radius of the penny-shaped crack
B_c	magnetic induction supported by the crack gap
B_r, B_z	magnetic induction components
D_c	electric displacement supported by the crack gap
D_r, D_z	electric displacement components
B_0, D_0, σ_0 or E_0, H_0, σ_0	magneto electro mechanical loading in poling direction
E_r, E_z	electric field components
H_r, H_z	magnetic field components
d_{11}, d_{33}	magnetoelectric constants
e_{15}, e_{31},e_{33}	piezoelectric constants
J_m	Bessel function of the first kind of order m
K_σ	mode I stress intensity factor
K_D	electric displacement intensity factor
K_B	magnetic induction intensity factor
$K_\sigma^* = (2/\pi)\sigma_0\sqrt{a}$	the classical result
r	radial coordinate
q_{15}, q_{31}, q_{33}	piezomagnetic constants
u_r, u_z	components of displacement vector
z	vertical coordinate
Δu_z	crack opening displacement
$\Delta\phi$	drop in electric potential across the crack
$\Delta\psi$	drop in magnetic potential across the crack
ε_{11}, ε_{33}	dielectric constants (permittivities)
$\varepsilon_c = \varepsilon_r\varepsilon_0$	dielectric constants of the material within the crack gap
$\varepsilon_0 = 8{,}85 \times 10^{-12}$ F/m	dielectric permittivity of air (or vacuum)
μ_{11}, μ_{33}	magnetic constants (permeabilities)
$\mu_c = \mu_r\mu_0$	magnetic permeability of the material within the crack gap
$\mu_0 = 4\pi \times 10^{-7}$ N/A^2	magnetic permeability of air (or vacuum)
ε_{rr}, $\varepsilon_{\theta\theta}$	components of stress tensor
ϕ	electric potential
ψ	magnetic potential
σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz}, σ_{rz}	components of stress tensor
ξ	Hankel parameter
λ_i ($i = 1,2,3,4$)	dimensionless roots appearing in general solution (eigenvalues defined by Eq. (2.7))

(Rogowski, 2011). Wang and Mai (2007) and Rogowski (2007) discussed the different electromagnetic boundary conditions on the crack- faces in PEMO-elastic materials. On the other hand, Zhong and Li (2007, 2008), Rogowski (2007) and Zhong (2009) have extended the semi-permeable crack-face electric boundary conditions proposed by Hao and Shen (1994) to analyze the PEMO-elastic fields induced by dielectric cracks. However, all of the studies considered only infinite body or plane layer problem (Zhong, 2009) and numerical procedures are used to obtained the results of approximate type. To the best of author knowledge, the penny-shaped crack problems for the layer and limited-permeable cracks have not been addressed yet, in exact form. Motivated by this the author of this paper investigates a PEMO-elastic layer, with an electrically and magnetically conducting crack under prescribed normal stress, electric displacement and magnetic induction boundary loadings or normal stress, electric and magnetic fields boundary conditions, to shown exact solution. Such solutions depend on a large number of material parameters, in our analysis it is seventeen, making any solution other than explicit analytical ones impractical.

The result could be of particular interest to the analysis and design of smart sensors and actuators constructed from magneto-electro-elastic composite laminates. Nowadays, electro-magneto-elastic coupled multiphase composite have wide range applications in science and engineering such as space planes, supersonic air planes, rockets, missiles nuclear fusion, reactors and submarines.

2. Basic equations in magnetoelectroelastic theory

The constitutive equations within the framework of linearly magnetoelectroelastic theory, in axially symmetric problem, can be written as