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# An intermingled fractal units model and method to predict permeability in porous rock

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ABSTRACT

For the calculation of the permeability of porous materials we have utilized an approach based on the description of the microstructure of the voids using fractal geometry. The fractal dimension of the microstructure has been calculated using porosimetric data obtained with the technique known as mercury intrusion porosimetry, subsequently simulating the experimental pore size distributions through the application of an Intermingled Fractal Units model based on unit type: the Sierpinski carpet. In this model a special analytical expression of the permeability has been studied from which a set of values that satisfactorily agree with those obtained from the experimental tests were derived. As reference material a porous rock (calcareous stone) has been considered.



## 1. Introduction

The study of the microstructure of porous materials is of considerable importance in order to understand the problems related to their physical–mechanical properties, durability and conservation. These characteristics vary further when the porous system of the materials is subject to the movement of water. This latter aspect generates dangerous situations due to those phenomena known as alternate actions of freeze–thaw, the salt crystallization (efflorescence, sub-florescence), the dissolution, the weakening of the bonds, the biological colonization, etc. For example Franzoni in (Franzoni & Sassoni, 2011) correlated stone microstructure (which depends on stone porosity and pore size distribution) to material degradation. One of the techniques that is still widely used for the determination of the porosity and its size distribution is known as the mercury intrusion porosimetry (MIP) into a porous material. The experimental data thus obtained contains a high number of information that can be used to establish the correlations between microstructure and properties. Given the fact the microstructure is in itself quite complex, its geometric description is necessary in order to be able to deeply understand its peculiarities. This descriptive and simulating aspect keeps on facing considerable difficulties due to both experimental techniques and the difficult reproducibility of the data obtained. Over the last few years, however, concepts and methods related to fractal geometry have been introduced. These concepts and methodologies have a remarkable affinity to treat very "irregular" figures. This "irregularity" is a distinctive character that is repeated in different dimensional scales (theoretically infinite) and is very well expressed by the concept of fractal dimension. For example, a surface that "curls" in space eventually occupies a certain volume, it can be considered as a figure which has a size greater than 2 and less than 3 (Falconer, 2003; Mandelbrot, 2004). Conversely, the ordinary geometrical dimensions are always represented by integers and they operatively play a very limited role. These concepts may appear as simple curiosity, but their remarkable versatility has allowed many various applications. For example, a number of studies that have found a correlation between the number of the

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islands of an archipelago or lakes or lunar craters to the respective surface can be used to correlate the number of pores with respect to their amplitude, in much the same way the number of pores can be correlated to their ray size, as it can be studied in (Shi, Xiao, Pan, & Yuan, 2006; Yu & Cheng, 2002; Yu & Liu, 2004). There are many papers that have recognized a connection between fractal geometry and the microstructure of porous materials. Some of them have brought to light the fractal characteristics of the roughness of the surfaces, the pore size distribution, as well as the profile of the pores, the fluid path, etc. In fact for example, Lange, Jennings, and Shah (1994) explored several image analysis techniques that provide insight into the nature of pore structure as observed in the backscattered electron images of polished sections. Liu and Ni (2009) evaluated fractal roughness effect of the two-dimensional micro Poiseuille gas flow, Winslow (1985) described an X-ray scattering technique to calculate the dimension of a fractal surface to demonstrate that the surface of hydrated cement paste is fractal in character, and has a large fractal dimension, Diamond (1999) have shown that pore systems of concrete may present fractal characteristics, meanwhile in Arandigoyen and Alvarez (2007) microstructure of blended mortars is studied taking into account porosity, pore size distribution and surface fractal dimension. Wang et al. (2012) proposed fractal analysis to quantify the complex and irregular pore structure. Eventually, Atzeni et al. measured fractal dimension, derived from pore size distribution, to correlate it with microstructure, its mechanical properties and permeability (Atzeni, Pia, & Sanna, 2010; Atzeni, Pia, Sanna, & Spanu, 2008a, 2008b). Much in the same way Pia et Al. studied a model for predicting thermal conductivity of lightweight concrete and advanced ceramics (Pia & Sanna, 2013a, 2013b). The fractality in the porous microstructure of the materials entails two different aspects, one connected to the pore size distribution ($D_f$) and another to their tortuosity ($D_t$). In this paper we are going to show how the fractal interpretation of the porosity can be related to the physical processes of diffusion of water. In particular an Intermingled Fractal Units model (denominated IFU) is presented, developed by varying some constructive aspects of the Sierpinski carpet. Simple fractals can be used effectively to describe pore size distributions which present a regular growth obtaining larger diameters. For this reason simple fractals are not efficient when it comes to describe very common structures which present one or more peaks in their distribution. But the use of more fractal units means that the IFU is able to effectively simulate the pore size distribution, the volume fractions of the voids as well as the geometry of the microstructure of non-fractal porous materials. After simulation, an analytical approach to obtain permeability values from the model will be presented. The importance of this procedure is that to calculate permeability pore size distribution each and every parameter has been taken into account. Moreover each parameter has been linked to its microstructure. Moreover fractal geometry consents to use simple analytical expressions. This type of modeling has also been used to predict thermal conductivity and mechanical resistance, so future development should research a unique model to predict different properties of the materials.

The theoretical treatment will be supported by an experimental verification conducted using a particular building material: sedimentary rock of Miocene age that in technical terms may be defined as calcareous stone, porous, easy workability, but poor durability if maintained in permanent contact with water (Atzeni, Sanna, & Spanu, 2006). This lithotype is, in terms of porosity and mechanical properties substantially equivalent to others that have been used, like this one, to build in the monuments of many areas of the Mediterranean (Rodolico, 1995).

## 2. The fractal pattern used to simulate a porous microstructure

The importance of this approach is due to the fact that it is particularly suitable for the description and analysis of complex systems. As far as porosity is concerned, Fractal Geometry, displays a well known series of "sponges", that regardless of their being originally very simple they call to mind the pore size distribution of the pores or the morphology of their outlines (as for example the Sierpinski carpet or its corresponding tridimensional model known as the Menger sponge) (Falconer, 2003; Mandelbrot, 2004). The development in the space of the scaling procedures (self-similarity, applied on no less than two/three orders of magnitude) allows processing the porosimetric data obtained with the well-known techniques of MIP or gas adsorption. Its applications can be found in scientific literature in large numbers and include traditional and advanced ceramic materials, sandstones, soils, but also cement materials. Recently it has also been possible to relate a number of relations enabling the effective and rational (not empirical) evaluation of physical quantities which are technologically relevant such as permeability to fluids: for example, Gimenez in (Gimenez, Perfect, Rawls, & Pachepsky, 1997) studied fractal and hydraulic properties of pore materials; Cai, Hu, Standnes, and You (2012)) presented a full analytical model for characterizing spontaneous imbibition of wetting liquid vertically into gas-saturated porous media including gravity over the whole imbibition process time frame; Turcio et al. (2013) proposed a fractal model for the effective permeability using the BMP (Bautista- Manero-Puig) model in porous media. The main assumptions of this analysis involve a bundle of tortuous capillaries whose size distribution and tortuosity follow the fractal scaling laws; Xu et al. in (Xu, Qiu, Yu, & Jiang, 2013) indicated that the geometrical parameters like porosity, fractal dimension for pore size distribution and tortuosity fractal dimension, have significant effect on the multiphase flow through unsaturated porous media. These predictions are compared to different models and with available experimental data; Berndt in (Berndt & Sevostianov, 2012) various models describing single phase fluid flow in a porous media with non-uniform distribution of pore sizes. So, in scientific literature there are several fractal models used to predict the physical properties of porous media. This reinforces the fact that the foundations on which fractal geometry is based are truly representative of the processes of formation or transformation of some materials. Sometimes, however, it is possible to find several purely analytical approaches that although arriving at the results correlated with the experimental data, lose the physical meaning of the process. That process is further ensured by a careful check of the geometric structure that has been considered. In this paper, a model based on a fractal known: namely, the Sierpinski carpet