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Short communication

Circulation cells in flow past a periodic wall

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ABSTRACT

We present a long wavelength model in order to derive a simple formula for the amplitude at the onset of circulation cells in flow past a wavy wall. The flow takes place between a wavy wall and a plane wall and is driven by moving the plane wall. There is no amplitude restriction but the Reynolds number is small and the wavelength is long. We find recirculation cells at an amplitude of about 3/10 the average gap width. This is confirmed by a Stokes flow calculation which can be done for all wavelengths.

At long wavelengths the circulation cells can be eliminated if, in addition to the moving wall, the flow is driven by gravity.

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1. A simple formula predicting recirculation

Studies of the flow of viscous fluids over wavy walls with emphasis on circulation cells have been presented by Pozrikidis (1987) and Scholle (2004). Our plan is to add to this two simple formulas predicting the onset of these cells at large and small wavelengths. Fig. 1 presents a sketch of the problem. The flow of a viscous fluid to the right is caused by the motion of the wall at $z = 0$. The wall at $z = 1 + A\cos(2\pi x)$ is not moving. The flow is steady and two dimensional.

Our aim is to find the value of A at which the flow direction is just at the point of reversing. We include a gravitational force acting to the right and obtain an unexpected result.

We first solve our problem in the long wavelength limit. A review and explanation of this approximation is presented by Oron, Davis, and Bankoff (1997). At short wavelengths we solve the Stokes equation in powers of A. We scale horizontal lengths by the wavelength, λ, vertical lengths by the average gap width, H, horizontal velocities by V, where V denotes a characteristic speed, and vertical velocities by $\frac{H}{\lambda}V$. Then denoting $\frac{gH^2}{\nu V}$ by G and $\frac{H^2}{\lambda\mu V}p$ by P we have, to zeroth order in $\frac{H}{\lambda}$, and in scaled variables,

$$\frac{dP}{dx} = \frac{\partial^2 v_x}{\partial z^2} + G \tag{1}$$

and

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \tag{2}$$

where $\frac{dP}{dx}$ is independent of z and we retain $\frac{dP}{dx}$ because we anticipate a large horizontal pressure gradient where the clearance is small.

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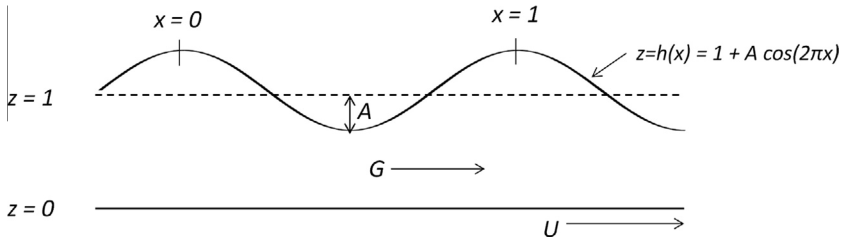

Fig. 1. A sketch of the problem.

The surfaces lie at $z = 0$ and $z = h(x) = 1 + A\cos(2\pi x)$, where v_x must be U at the first, zero at the second, and where v_z vanishes at both surfaces. Thus we have

$$v_x = \frac{1}{2}\left(\frac{dP}{dx} - G\right)(z^2 - zh) + U\left(1 - \frac{z}{h}\right) \tag{3}$$

$$v_z = -\frac{1}{2}\frac{d^2P}{dx^2}\left(\frac{z^3}{3} - \frac{z^2h}{2}\right) + \frac{1}{4}\left(\frac{dP}{dx} - G\right)z^2\frac{dh}{dx} - \frac{1}{2}U\frac{z^2}{h^2}\frac{dh}{dx} \tag{4}$$

and

$$Q = \int_0^h v_x dz = \frac{1}{12}\left(-\frac{dP}{dx} + G\right)h^3 + \frac{1}{2}hU \tag{5}$$

and henceforth we write $\Pi(x)$ in place of $-\frac{dP}{dx}$.

To find $\Pi(x)$ we set $\frac{dQ}{dx}$ to zero and obtain

$$\frac{d}{dx}(\Pi + G) + \frac{3}{h}\frac{dh}{dx}(\Pi + G) + \frac{6}{h^3}\frac{dh}{dx}U = 0 \tag{6}$$

whence we have

$$\Pi + G = [\Pi(x = 0) + G]\frac{h^3(x = 0)}{h^3} - \frac{6U}{h^3}[h - h(x = 0)] \tag{7}$$

Now P is periodic in x having period 1, hence we have $\int_0^1 \Pi dx = 0$ and integrating Eq. (7) over a period, we obtain

$$[\Pi(x = 0) + G]h^3(x = 0) = \frac{G + 6U\int_0^1 \frac{h - h(x=0)}{h^3}dx}{\int_0^1 \frac{1}{h^3}dx} \tag{8}$$

whereupon we have

$$\Pi + G = \frac{1}{h^3}\frac{G + 6U\int_0^1 \frac{1}{h^2}dx}{\int_0^1 \frac{1}{h^3}dx} - \frac{6U}{h^2} \tag{9}$$

and again

$$v_x = -\frac{1}{2}(\Pi + G)(z^2 - zh) + U\left(1 - \frac{z}{h}\right) \tag{10}$$

To find the value of A corresponding to the onset of circulation cells, we notice that the first sign of recirculation ought to appear at $x = 0$, cf. Fig. 1, and corresponds to the tangential stress at the wavy wall, i.e., $\frac{dv_x}{dz}$ in the long wavelength approximation, vanishing there. Thus we have

$$\frac{dv_x}{dz}(x, z = h) = -\frac{1}{2}(\Pi(x) + G)h - \frac{U}{h} \tag{11}$$

and by setting this to zero we obtain

$$\frac{G}{2U} = 2h\int_0^1 \frac{1}{h^3}dx - 3\int_0^1 \frac{1}{h^2}dx \tag{12}$$

which is to be evaluated at $x = 0$ where $h = 1 + A$. The right hand side depends only on A and we present the RHS versus A in Fig. 2. This is our long wavelength prediction.

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