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Interfacial boundary conditions between a free domain and thin porous layers for non-Newtonian fluid flows

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ABSTRACT

A non-Newtonian fluid flows in a free domain and in a periodically perforated thin layer which are connected through a permeable interface. Two scales are present in the porous layer: one associated to the periodicity of the distribution of the channels which is associated to the thinness of the layer and the other to the diameter of these channels. Using Γ-convergence and two-scale convergence methods, we derive boundary conditions of Beavers–Joseph–Saffman type on the permeable interface between the free domain and the thin layer.

1. Introduction

In 1957, Truesdell initiated in Truesdell (1957) the so-called mixture theory which provides a systematic framework for the study of the thermomechanics of interacting continua, even when interconversion takes place between the constituents of the mixture. He postulated the balance equations of mass, momentum and energy for each constituent of a mixture. Later, Truesdell described in Truesdell (1962) the mathematical theory of the diffusion in a mixture through four different approaches:

1. the kinematical one, leading to Fick's equation of diffusion,
2. the hydrodynamical Maxwell–Stefan equations of motion for the different constituents of a mixture of fluids,
3. the kinetic Maxwell–Chapman–Enskog formulas in a mixture of dilute monatomic gases,
4. the thermodynamic approach, which is suitable for the diffusive flux in more general fluid mixtures.

Following the pioneering works of Truesdell, several authors (see Atkin & Craine, 1976a,b; Bowen, 1967; Green and Naghdi, 1969; Kelly, 1964, for example) have developed the formulation of the continuum theory of mixtures, establishing its rigorous mathematical foundation. The application of the Clausius–Duhem inequality in the derivation of restrictions on the constitutive equations of mixtures of chemically reacting, linearly viscous and compressible fluids, has been described in the first book (Samohyl, 1987) on mixture theory. The author here proved that the partial thermodynamic quantities may be deduced from the dependence of mixture properties on its composition.

The mixture theory methodology may be interpreted, in some sense, as a homogenization approach regarding each component as a single continuum and assuming that, at each time, every point in space is occupied by a particle belonging to one

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component of the mixture (Truesdell, 1984). The basic theory of mixtures has been discussed in Rajagopal and Tao (1995), the authors focusing on the treatment of diffusion problems for a fluid through a nonlinear elastic solid, of wave propagation problems, and of mixtures of fluids and solid particles. The insensitivity of the flow of a fluid through a porous elastic solid undergoing large deformation is discussed in Prasad and Rajagopal (2006).

One of the outstanding issues in mixture theory, still much debated, is the specification of boundary conditions which have to be imposed on the interfaces between the constituents. This is addressed in Rajagopal, Wineman, and Gandhi (1986) and Tao and Rajagopal (1995), among others, for fluid flows through non-linear elastic solids such as rubber, and for general mixtures, through the use of a thermodynamic principle.

The main purpose of the present work is to derive appropriate boundary conditions which have to be imposed for non-Newtonian fluid flows at a permeable wall between a free part and a thin porous layer. The study of non-Newtonian flows through porous media is important in various branches of industry dealing for example with polymer melts, solutions, heavy oils and other complex fluids (see Bourgeat, Gipouloux, & Marusic-Paloka, 2003). Fluid flows through a domain consisting of a free region and of a porous medium occur in a wide variety of domains such as biomechanics, medicine and natural phenomena. For instance, such fluid flows happen in membrane filtration (see Ripperger & Altmann, 2002), or when a viscous fluid flows over a bed of solid particles during the solidification of multi-component melts, the solid and the fluid being separated by a layer of mixed phase, called mushy layer, which continuously evolves because of internal solidification and local dissolution processes (see Le Bars & Worster, 2006; Worster, 1997). Such fluid flows may also occur in porous living tissues, which allow blood flows to supply nutrients to the neighboring cells of the tissue (see Rubin & Bodner, 2002).

The interactions between a free fluid flow and a fluid flow in a porous medium that occur when the two domains are connected give rise to alterations of the flow characteristics in thin layers surrounding the interface between the two regions. The lubrication process associated to these interactions provided the impetus for the original experimental study of Beavers and Joseph (1967). Numerous subsequent studies, among which (Brillard, 1986; Brillard, El Amrani, & El Jarroudi, 2013; Cieszko & Kubik, 1999; dell'Isola, Madeo, & Seppecher, 2009; El Jarroudi, 2010; Ene & Sanchez-Palencia, 1975; Jäger & Mikelic, 1996; Jäger & Mikelic, 2000; Jäger & Mikelic, 2009; Saffman, 1971), have described this phenomenon. Through their experimental devices, Beavers and Joseph (1967) determined the boundary conditions at the permeable interface, which take the form

$$\frac{\partial v_\tau^1}{\partial n} = \gamma K^{-1/2}(v_\tau^1 - v_\tau^2), \tag{1}$$

where n is the outer unit normal, v_τ^1 is the tangential component of the fluid velocity in the free region, v_τ^2 is the tangential component of the Darcy velocity of the fluid in the porous medium, K is the permeability of the porous medium and γ is some slippage coefficient. This model has been then theoretically studied by Saffman in Saffman (1971), who also proved that the term v_τ^2 can be neglected.

In the present paper, we consider non-Newtonian fluid flows and especially the case of a share-dependent viscosity that is of a constitutive equation obeying the power law

$$\begin{cases} \sigma = -pId_{\mathbb{R}^3} + \mu|\mathbf{D}u|^{r-2}\mathbf{D}u, \\ \mathbf{D}u = \frac{1}{2}(\nabla u + \nabla u^t), \end{cases} \tag{2}$$

where σ is the Cauchy stress (including the case of a Newtonian flow for $r = 2$), p is the pressure, $Id_{\mathbb{R}^3}$ is the identity matrix on $\mathbb{R}^3$, u is the fluid velocity, $\mathbf{D}u$ is the symmetric part of the strain tensor, ∇u is the gradient velocity tensor, μ is the zero-shear-rate viscosity or Newtonian viscosity of the fluid and $r \in]1,2]$. The parameter r may depend on the temperature (see Antontsev & Rodrigues, 2006) or on the pressure (see Málek, Nečas, & Rajagopal, 2002) in the fluid flows. The constitutive relation (2) is frame indifferent as it involves the symmetric part $\mathbf{D}u$ of the strain tensor.

For the description of the behavior of this mixture on the permeable interface between the free part and the porous layer, we will use the Γ-convergence and two-scale convergence methods. We will derive the following interfacial boundary conditions

$$\begin{cases} |\mathbf{D}u|^{r-2}\mathbf{D}u \cdot n = \gamma K_r^{bl}|u_\tau - v_\tau|^{r-2}(u_\tau - v_\tau), \\ u_n = 0, \end{cases} \tag{3}$$

where $\mathbf{D}u \cdot n = (\mathbf{D}_{1j}n_j, \mathbf{D}_{2j}n_j, \mathbf{D}_{3j}n_j)^T$, K_r^{bl} is a boundary layer tensor whose expression is given in (19), γ is a positive constant corresponding to the thickness of the boundary layer which takes place in the neighborhood of the permeable interface, u_τ (resp. u_n) is the tangential (resp. normal) component of the non-Newtonian fluid velocity in the free region and v_τ is the surfacic Darcy velocity of the fluid in the thin porous layer. In the case where $r = 2$ (see Brillard et al., 2013), the interfacial boundary condition (3) is of Beavers–Joseph–Saffman type, compare to (1). In the present paper, we consider a more general situation, taking $r \in]1,2]$ and the interfacial boundary condition (3) may be of nonlinear type.

We first describe the fluid flows either in the free region or in the thin perforated layer. The description of the asymptotic behavior of the fluid flows starts with the proof of uniform estimates on the velocity fields of the fluid flows which lead to the definition of an appropriate topology, then with the construction of test-functions which allow to pass to the limit in the original problem. The computations are given with lengthy details, although some technical results are postponed to an Appendix.